MATH/GPHS 322, 323 2014

Module on Fluid Flow in Earth Systems, Assignment 1, first week of module. Due date is 5 PM on Friday 12 September in assignment box next to SGEES admin office, 3rd floor, Cotton Building

Reading: Turcotte & Schubert (either edition). The Course notes available for purchase at the bookstore contains all the material you need from Edition 2.
This week: Chapters 6-1 through 6-6. Problems:
6-1 through 6-9, which are reprinted below. Note that some of these problems are very easy, requiring merely plugging in numbers. However, I find that they are worthwhile because they give a better understanding of the physical meaning of the equations. Similarly, the mathematics in this chapter is simple and concentrates on one and two-dimensional problems that have easily tractable analytic solutions. The full 3D treatment of fluid flow is best analysed with computer analysis, as will be done in the last (project) part of the course. Note also that the answers to some of the problems are right after the Appendices. Of course, you must show your work to get credit.

Marking: You do not need to necessarily get the correct answer to get some credit. However, you also will not necessarily get credit for getting the correct answer if you don't show how you did it. To get full credit (10 marks for each problem), you must do the following:
1) Draw a figure explaining the problem, with the coordinate system and any symbols explained. (2 marks)
2) Any equation you use must be either referenced, e.g., with the equation number from Turcotte & Schubert, stating which edition (the edition can be stated once at the start of the assignment) or else derived from a preceding equation with any non-obvious steps explained. (1 mark)
3) Highlight the answer at the end in some fashion, e.g., underline or box or label ANSWER. (1 mark)
The last 6 marks are for the proper working of the problem.

There are three assignment sets for the analytic section, taken mainly from problems in Turcotte and Schubert. There are also three assignment sets from the computer section. Each assignment set is worth equal marks. Within the analytic section, each problem in an assignment set is worth equal marks.

On each assignment set, marking will be done carefully on half of the problems, chosen randomly after they are turned in. The others will be looked at for completeness but not marked.

6-1: Show that the mean velocity in the channel just considered in section 6-2 (i.e., where $u=u_0$ at the top of the channel, $u=0$ at the bottom and $dP/dx \neq 0$) is given by $\bar{u} = -\frac{h^2}{12\mu}\frac{dp}{dx} + \frac{u_0}{2}$.

6-2: Derive a general expression for the shear stress τ at any location y in the channel. What are the simplified forms of τ for Couette flow and for the case of $u_0=0$? In each case, where is τ a maximum and minimum, and what values does it have there?

6-3: Find the point in the channel at which the velocity is a maximum.

6-4: Consider the steady, unidirectional flow of a viscous fluid down the upper face of an inclined plane. Assume that the flow occurs in a layer of constant thickness, *h*, as shown in Figure 6-3. In the figure, *y* is the coordinate measured perpendicular to the inclined plane (*y*=*h* is the surface of the plane), α is the inclination of the plane to the horizontal, and *g* is the acceleration of gravity.

First show that $\frac{d\tau}{dy} = -\rho g \sin\alpha,$

Then apply the no-slip condition at $y=h$ and the free surface condition, $\tau=0$, at $y=0$ to show that the velocity profile is given by

$$u = \frac{\rho g \sin\alpha}{2\mu}(h^2 - y^2),$$

What is the mean velocity of the layer? What is the thickness of a layer whose rate of flow down the incline (per unit width in the direction perpendicular to the plane in Figure 6-3) is Q?

6-5: For an asthenosphere with a viscosity $\mu=4\times10^{19}$ Pa s and a thickness h=200 km, what is the shear stress on the base of the lithosphere if there is no counterflow ($\frac{\partial p}{\partial x} = 0$; as is now considered to be the case)? Assume u_0=50 mm/yr and that the base of the asthenosphere has zero velocity.

6-6: Assume that the base stress obtained in Problem 6-5 is acting on 6000 km of lithosphere with a thickness of 100 km. What tensional stress in the lithosphere (h_L=100 km) must be applied at a trench to overcome this basal drag?

6-7: Determine the Reynolds number for the asthenospheric flow considered in Problem 6-5. Base the Reynolds number on the thickness of the flowing layer and the mean velocity (u_0=50 mm/yr and ρ=3200 kg m^{-3}). This problem illustrates that the viscosity of mantle rock is so high that the Reynolds number is generally small.

6-8: A spring has a flow of 100 litres per minute. The entrance to the spring lies 2 km away from the outlet and 50 m above it. If the aquifer supplying the spring is modelled according to Figure 6-9, find its cross-sectional radius. What is the average velocity? Is the flow laminar or turbulent?

6-9: Determine the rate at which magma flows up a two-dimensional channel of width d under the buoyant pressure gradient $-(\rho_s-\rho_l)g$. Assume laminar flow.