MATH/GPHS 322, 323 2014

Module on Fluid Flow in Earth Systems, Assignment 3. Due 5 PM Friday 26 Sept.

Reading: Turcotte & Schubert, the following sections:

Marking: You do not need to necessarily get the correct answer to get some credit. However, you also will not necessarily get credit for getting the correct answer if you don't show how you did it. To get full credit (10 marks for each problem), you must do the following:
1) Draw a figure explaining the problem, with the coordinate system and any symbols explained. (2 marks)
2) Any equation you use must be either referenced, e.g., with the equation number from Turcotte & Schubert, stating which edition (the edition can be stated once at the start of the assignment) or else derived from a preceding equation with any non-obvious steps explained. (1 mark)
3) Highlight the answer at the end in some fashion, e.g., underline or box or label ANSWER. (1 mark)
The last 6 marks are for the proper working of the problem.

Erratum:

Turcotte & Schubert, Edition 2 has the following misprints:

p. 273 the equation in the middle left is based on Eq. 4-116 (*not* 4-117)
Eq. 6-346 should have ph replaced by ρH, i.e., the density times the heat production per unit mass.
Problem 6-32 p. 280 should be referring to equations 6-342 and 6-343, (*not* 6-341 and 6-342).
Fig. 6-42 should replace T_c=T_1

Assignment:

Section 6-15. Plume Heads and Tails—this is only in the second edition. It helps better to consolidate Ch. 4 and Section 6-14 and to prepare you for the next chapters. Read it and do:

1. Problem 6-24. Determine the radius of the plume conduit, the volume heat flux, the mean ascent velocity, and the plume head volume for the Azores plume. Assume that T_p-T_1=200 K, α_v=3x10^{-5} K^{-1}, μ_p=10^{19} Pa s, ρ_m=3300 kg m^{-3}, μ_m=10^{21} Pa s, and c_p=1.25 kJ kg^{-1}K^{-1}.

Section 6-15 Ed. 1 or 6-16, Ed. 2, Pipe Flow with Heat addition,

2. Problem 6-24 edition 1 or 6-26 Edition 2: Consider unidirectional flow driven by a constant horizontal pressure gradient through a channel with stationary plane parallel walls, as discussed in Section 6-2. Determine the temperature distribution in the channel, the wall heat flux, the heat transfer coefficient, and the Nusselt number by assuming, as in the pipe flow problem, that the temperature of both walls and the fluid varies linearly with distance x along the channel. You will need the form of the temperature in two dimensions that balances horizontal heat advection against vertical heat conduction, as given in Equation 4-156: $U\frac{\partial T}{\partial x}=\kappa\frac{\partial^2 T}{\partial y^2}$. You will need to understand a little bit about Heat Flow to follow the chapter, so I am including Ch. 4 for you to just study mostly the relevant section (Section 4-13). Hint: The answer in the appendix of the book left out a factor of dp/dx and they call the height of the channel d. You may also find it easiest to start the problem by switching coordinate systems so that y'=y-d/2.

Section 6-17 Ed. 1 or 6-18 Ed. 2, Thermal Convection, No Problems

Section 6-18 Ed. 1 or 6-19 Ed. 2, Linear Stability Analysis,

3. Problems 6-27, Ed. 1 or 6-29, Ed. 2. Estimate the values of the Rayleigh numbers for the mantles of Mercury, Venus, Mars and the Moon. Assume heat is generated internally at the same rate it is produced in the Earth. Use the same values for μ, k, κ, and α_v as used above for the Earth's mantle. Obtain appropriate values of ρ_0, g, and b from the appendices or the internet.
4. 6-28, Ed. 1 or 6-30, Ed. 2: Calculate the exact minimum and maximum values of the wavelength for disturbances that are convectively unstable at Ra=2000. Consider a fluid layer heated from below with free-slip boundary conditions. Hint : Sometimes a numerical solution to a problem, e.g. through a spreadsheet, is easier than an analytic solution. For instance, to find the minimum of a function, you could calculate the function at a lot of points using a spreadsheet instead of taking its analytic derivative. This is a perfectly valid method of solving a problem.
5. 6-29, Ed. 1 or 6-31, Ed. 2 but modified as follows: (Note: the equation numbers below assume Edition 2) The section outlines the steps needed to carry out the linear stability problem for the onset of convection in a layer of fluid heated from below. You will formulate the linear stability problem for the onset of convection in a layer of fluid heated from within (i.e., in the case of the Earth, if there is radioactivity included). Assume that the boundaries are stress-free. Take the upper boundary to be isothermal and the lower boundary to be insulating. Draw a diagram of the problem. Show how equations 6-293 in Edition 2 should be modified. Continue showing how to modify the rest of the equations from 6-294 through 6-302. Formulate the equations to incorporate the new boundary conditions, so that you can come up with equations to replace 6-303 through 6-306. Introduce a stream function and carry through the first step to determine the new analogue to equations 6-307 through 6-309. Discuss the differences between your formulation and the formulation for fluid heated from below. Without solving the rest of the equations, try to give some insights into why the forms of the Rayleigh number in equation 6-324 is different to that in equation 6-316.

Section 6-21 Ed. 1 or 6-23, Ed. 2 Heating by viscous dissipation,

6. Problems 6-35 Ed. 1 or 6-37 Ed 2: Show that half of the frictionally generated heat flows out of the lower boundary of the channel in the Couette flow example in this section.

7. Problem 6-36 Ed. 1 or 6-38 Ed. 2: Consider frictional heating in a Couette flow with an isothermal upper boundary and an insulated lower boundary. Determine the temperature profile in the channel and the excess upward heat flow at the upper boundary due to the shear heating. What is the temperature of the lower boundary as a consequence of the frictional heating? Compare the temperature rise across this channel with the maximum temperature rise in a channel with equal wall temperatures.