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MATH 321/322/323 Applied Mathematics T1 and T2 2013

Module on Special Relativity: Assignment 5

This fifth assignment covers chapters 7 and 8:
— "Mom-energy", "Collide, create, annihilate".
(Also, check out appropriate pats of the notes.)

1. **One step beyond Newton:**

   [The mathematics here is very elementary.]

   The relativistic formula for total energy is (in SI units)

   $$E = \frac{m_0 c^2}{\sqrt{1 - v^2/c^2}}.$$

   In the book you saw, using the binomial expansion, that

   $$(1+x)^n = 1 + nx + O[x^2].$$

   Therefore, at low speeds

   $$E = m_0 c^2 + \frac{1}{2} m_0 v^2 + O[v^4],$$

   so that the total energy can be approximated as "rest energy" $m_0 c^2$, plus Newtonian kinetic energy $\frac{1}{2} m_0 v^2$, plus a special relativity correction.

   Find or derive a higher-order version of the binomial expansion

   $$(1+x)^n = 1 + nx + ... + O[x^3]$$

   What is the missing term?

   Hence find the first relativistic correction to the kinetic energy using the formula

   $$E = m_0 c^2 + \frac{1}{2} m_0 v^2 + ... + O[v^6]$$

   What is the missing term?

2. **Many steps beyond Newton:**

   [Now the mathematics is a little more subtle.]

   Expand the formula for total relativistic energy

   $$E(v) = \frac{m_0 c^2}{\sqrt{1 - v^2/c^2}}$$

   as a Taylor series in v:

   $$E(v) = \sum_{n=0}^{\infty} a_n \, v^n.$$

   Evaluate the general Taylor coefficient a_n.

   (There is a much smarter way of doing this than brute force differentiation. Think!)

   What is the radius of convergence for the Taylor series?

   (You do not need to remember anything more subtle than the "ratio test" you saw in 2nd year Calculus to answer this. Think!)

   Can you come up with a good physics reason for the mathematical answer you get?

3. Do problem 8-4 on page 254 (the "measurement of photon energy" problem).

4. Do problem 8-6 on page 258 (the "gravitational redshift" problem).

5. Do problem 8-8 on page 259 (the "nuclear excitation" problem).

6. Do problem 8-10 on page 259 (the "photon integrity" problem).

7. Do problem 8-12 on page 259 (the "photo-production of an electron-positron pair by two photons" problem).

8. Do problem 8-16 on page 261 (the "creation of proton-antiproton pair by an electron" problem).

9. Do problem 8-18 on page 263 (the "Doppler shift along the x-direction" problem).

10. Do problem 8-26 on page 264 (the "$E = mc^2$ from the Doppler shift" problem).

- You may note that the textbook provides *many* more problems that I could have set for these chapters; and some of *those* problems are considerably harder than the ones above.
- The problems above are actually relatively straightforward; and should not cause too much trouble if you keep your wits about you.

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