MATH 322/323 Module 1 Cartesian Tensors Mar 5 – May 5 2014

Timetable

Week	1	2	3	4	5	6	7
Start	Mar 3	Mar 10	Mar 17 Assignment 1 due	Mar 24 Assignment 2 due	March 31	Apr 7 Assignment 3 due	April 14
Mon 14:10-15:00	Intro lecture	L3	L5	L7	L9	Spare	L11
Tues 14:10-15:00	L1 Assignment 1 set	L4 Assignment 2 set	L6 Assignment 3 set	L8 Assignment 4 set	L10	spare	T6
Weds 14:10-15:00	L2	Spare	Spare	Spare	Spare	Spare	Spare
Tutorial Fri 14:10-15:00	T1	T2	T3	T4	T5	Spare	Assignment 4 due Thursday 17 April

Assignments and tutorial exercises

All assignments due 5pm on day of week shown.

Essay due 5pm Monday 5 May

Assessment Summary

Assignment 1	20%	Index notation; Rotational transformations; Euler vector
Assignment 2	20%	Prove Kronecker is a tensor; lead rubber bearing, stress force across a plane
Assignment 3	20%	Strain gauges – principal axes, simple shear
Assignment 4	20%	Hooke's Law, tensor calculus
Essay	20%	**due 5 May.**

Tutorial One 7 March

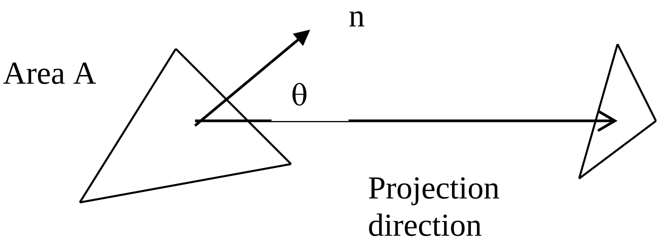
Revision – vectors and linear algebra. Try these yourself before tutorial time.

1. Let **a** be the position vector of a given point $(x_{10}, x_{20}, x_{30})^T$ and **r** be the position vector of any point $(x_1, x_2, x_3)^T$. Describe the locus of **r** if:

a: $|\mathbf{r} - \mathbf{a}| = 3$; b: $(\mathbf{r} - \mathbf{a}) \cdot \mathbf{a} = 0$; c: $(\mathbf{r} - \mathbf{a}) \cdot \mathbf{r} = 0$

2. a: Show that the area of a triangle formed by two vectors **a** and **b** is ½ $|\mathbf{a} \times \mathbf{b}|$

b: Hence show that he projected area of a triangle = (area of the triangle) x cosine of the angle between the normal to the triangle and the projection direction; ie:



Area = A cos θ

NB this is a very well known result, but one that is hardly ever proved!

3. If <u>**a**</u> and <u>**b**</u> are distinct vectors, construct a RH Cartesian set of axes where x_1 is normal to the plane of <u>**a**</u> and <u>**b**</u> and x_2 and x_3 are any two vectors in the plane of <u>**a**</u> and <u>**b**</u>.

4. Attempt to find a non-trivial solution to Ax = 0 -

(i) For A =

$$\begin{bmatrix} 3 & 4 & 5 \\ 2 & -1 & 2 \\ -1 & -5 & -3 \end{bmatrix}$$

(ii) for A =

$$\begin{bmatrix} 3 & 4 & 0 \\ 2 & -1 & 0 \\ 0 & 11 & 1 \end{bmatrix}$$

Changing axes

5. $\mathbf{u_1}$ and $\mathbf{u_2}$ are two mutually perpendicular vectors, which are used to construct a new coordinate system. **y** is a vector described in the old coordinate system. What are the coordinates of the endpoint of **y** in the new system?

6. Construct transformation matrices A for giving the coordinates of a vector <u>**p**</u> in a new coordinate system, using the convention
<u>**p**</u> (new) = A^T <u>**p**</u> (old), for:

(a) Rotation through 90^o about **x2** axis,

(b) Rotation through 45^o about **x2**, followed by rotation through 45^o about (new) **x1,**

In each case:

(i) verify that your transformation works by applying it to a suitable test vector e.g. one of the coordinate axes.

(ii) find $|A|$ (determinant A).

MATH 322/323 Cartesian Tensors

Assignment 1 ***due Monday 17 March.***

Marking: You do not need to necessarily get the correct answer to get some credit. However, you also will not necessarily get credit for getting the correct answer if you don't show how you did it. To get full credit (10 marks for each problem), you must do the following:

1) Draw a figure explaining the problem, with the coordinate system and any symbols explained. (2 marks-note, there are a few cases where no figure is required.)

2) Any equation you use must be either referenced, e.g., with the equation number from the notes or from another book (referenced) or else derived from a preceding equation with any non-obvious steps explained. (1 mark)

3) Highlight the answer at the end in some fashion, e.g., underline or box or label ANSWER. (1 mark)

The last 6 marks are for the proper working of the problem.

(1) *Ascertain whether the following are valid index set equations*. If any are invalid, re-write the RHS or the LHS to make them valid. (no figure necessary)

(a) $a_{ij} = b_{ij} c_{jk}$
(b) $a_{ijj} = b_{ik} c_k$
(c) $a_i = b_{ij} c_{jk} b_{ik}$

(2) $\underline{\mathbf{w}} \in R_k$, $\underline{\mathbf{v}} \in R_n$ and A, X, and Y are (real) matrices: A is n by k, Y is k by p, Z is k by p, and X is n by p . ^{T} denotes transpose. *Write valid index set equations for (no figure necessary)*:

(a) $\underline{\mathbf{w}}^T = \underline{\mathbf{v}}^T A$
(b) $X = A\,Y$
(c) $Z = A^T X$

(3) *Write in matrix notation (No figure necessary)*:

(a) $a_{ik} = b_{ij} c_{jn} d_{nk}$
(b) $a_{ik} = b_{ij} c_{nj} d_{nk}$
(c) $a_{ik} = d_{nk} c_{jn} b_{ij}$

(4) Construct transformation matrices A, using the convention
$\underline{\mathbf{p}}$ (new) = A^T $\underline{\mathbf{p}}$ (old), for:

(a) Rotation through 180° about **x3** axis,
(b) Rotation through 45° about **x1** axis,
(c) Rotation through θ about the **x2** axis,
(d) Rotation through 90° about **x1**, followed by rotation through 45° about (new) **x2,**
(e) Reflection in the **x1, x3** plane.

In each case:
(i) verify that your transformation works by applying it to a suitable test vector e.g. one of the coordinate axes.
(ii) find $|A|$ (determinant A).

Method: write down the unit vectors describing the new axes, and use the lecture results to construct A.
Hint: sketch the old and new axes for each rotation.

Question (5) is on the next page.

(5) Euler's theorem

A rigid body has a Cartesian coordinate system embedded into it. The (unit) vectors describing the axes are:

α_1	α_2	α_3
0.7229	0.5883	0.3623
-0.5623	0.1961	0.8034
0.4016	-0.7845	0.4726

It rotates about the origin to a location where the axes are now described by the vectors:

β_1	β_2	β_3
-0.7071	0.3015	0.6396
0.7071	0.3015	0.6396
0	0.9046	-0.4264

(a) Solve Chapter 1 equations (4) (for i = 1, 2) in the proof of Euler's theorem to find the axis (unit) vector $\underline{\mathbf{x}}_{\mathbf{E}}$ for the rotation. *Verify* that your vector satisfies eqn (4-3).

Hint: Since only two components of $\underline{\mathbf{x}}_{\mathbf{E}}$ are independent, solve for the ratios

$\underline{\mathbf{x}}_{\mathbf{E}\,1} / \underline{\mathbf{x}}_{\mathbf{E}\,3}$ and $\underline{\mathbf{x}}_{\mathbf{E}\,2} / \underline{\mathbf{x}}_{\mathbf{E}\,3}$.

(b) Find the angle through which the rigid body was rotated, as follows:

(i) Find (any) unit vector $\underline{\mathbf{r}}$ at right angles to $\underline{\mathbf{x}}_{\mathbf{E}}$.
(ii) Use the equations given in lectures for transforming between coordinate systems to find the description of $\underline{\mathbf{r}}$ in the α and β systems, = $\underline{\mathbf{r}}^{\alpha}$ and $\underline{\mathbf{r}}^{\beta}$.
(ii) The scalar product of $\underline{\mathbf{r}}^{\alpha}$ and $\underline{\mathbf{r}}^{\beta}$ gives the cosine of the rotation angle.

You must use a programming language (Maple, MatLab) or Excel (or similar freeware) for the arithmetic.