MATH 322/323 Module 1 Cartesian Tensors Mar 5 – May 5 2014

Timetable

Week	1	2	3	4	5	6	7
Start	Mar 3	Mar 10	Mar 17 Assignment 1 due	Mar 24 Assignment 2 due	March 31	Apr 7 Assignment 3 due	April 14
Mon 14:10-15:00	Intro lecture	L3	L5	L7	L9	Spare	L11
Tues 14:10-15:00	L1 Assignment 1 set	L4 Assignment 2 set	L6 Assignment 3 set	L8 Assignment 4 set	L10	spare	T6
Weds 14:10-15:00	L2	Spare	Spare	Spare	Spare	Spare	Spare
Tutorial Fri 14:10-15:00	T1	T2	T3	T4	T5	Spare	Assignment 4 due Thursday 17 April

Assignments and tutorial exercises

All assignments due 5pm on day of week shown.

Essay due 5pm Monday 5 May

Assessment Summary

Assignment 1	20%	Index notation; Rotational transformations; Euler vector
Assignment 2	20%	Prove Kronecker is a tensor; lead rubber bearing, stress force across a plane
Assignment 3	20%	Strain gauges – principal axes, simple shear
Assignment 4	20%	Hooke's Law, tensor calculus
Essay	20%	**due 5 May.**

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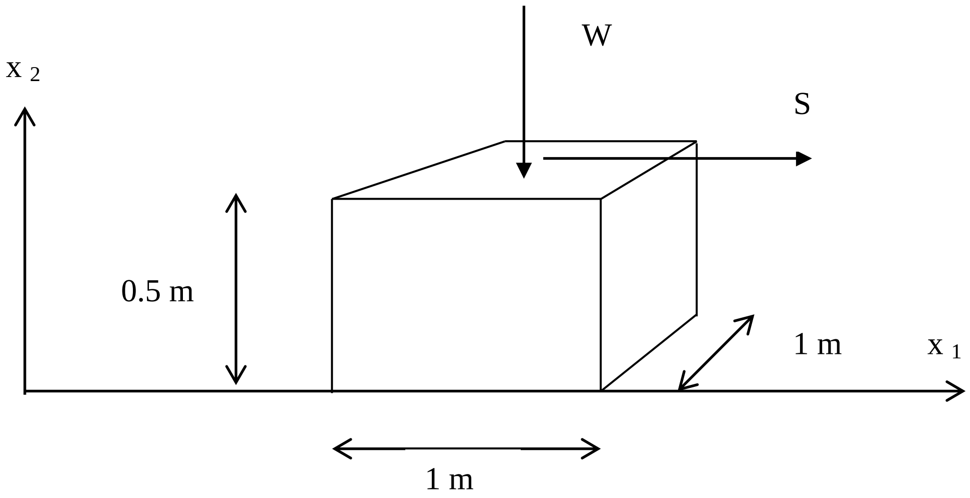
Assignment 2 *due Monday 24 March;* **Notes Chapter 1 and start of Chapter 2.**

(1) Show formally that the Kronecker Delta δ_{ij} is a tensor; i.e. for any (orthogonal) transformation of the coordinate system given by a_{pq}, show that δ_{ij} satisfies:

$$\delta'_{ij} = a_{ip} a_{jq} \delta_{pq}$$

(2) Show formally that the index set defined by $x_j = 1$, $j = 1,2,3$ for *every* set of Cartesian coordinate axes, is *not* a tensor. NB if it fails the test for any one transformation, it is not a tensor.

(3) Lead rubber bearings for damping earthquake motions have been fitted to the columns of the Rankine Brown building and Te Papa. They are tested by applying a load W equal to the share of the weight of the building and then applying a shear force S to simulate earthquake forces:

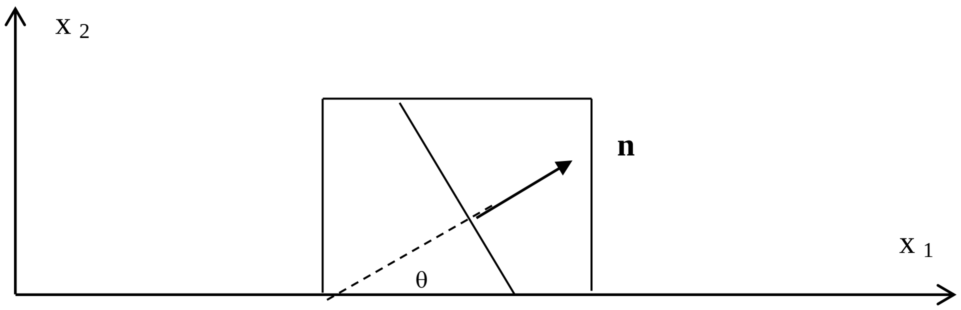


A lead rubber bearing is modelled as a homogeneous cuboid 1m x 1m bearing area by 0.5 m high. If W = 50 MN and S = 10 MN,

(i) What additional forces must be applied to keep the block in equilibrium? (i.e. stop the block rotating or accelerating)?

(ii) Write down the stress tensor for the bearing.

(iii) Find the Principal Axes of the stress tensor and the Principal Stresses.



(iv) Find the stress force F per unit area inside the block across a plane with its normal in the $x_1 x_2$ plane, making an angle θ with the x_1 axis i.e. $\mathbf{n} = (\cos\theta, \sin\theta, 0)$.

(v) Write down expressions for the Normal, N, and total Shear, S_T, components of F, and find the orientation(s) of the plane which makes the ***magnitude*** of each, separately, a maximum.

(vi) Hence find the maximum Shear and Normal stresses in the block.

Hints:

1. To find the shear force, find the direction of the total Shear force, and take the scalar product with F.
2. Write the expressions for $|N|$ and $|S_T|$ in terms of 2θ before differentiating to find the maxima.

Tutorial Two 14 March AND Tutorial Three 21 March

(0) Complete any questions from Tutorial one.

(1) Construct transformation matrices A for giving the coordinates of a vector **p** in a new coordinate system, using the convention
p (new) = A^T **p** (old), for:

(a) Rotation through θ^o about **x1** axis,
(b) Rotation through θ^o about **x2** axis,
(c) Rotation through θ^o about **x3** axis

2. Show formally that the Alternating Tensor ε_{ijk} is a tensor; i.e. for any (orthogonal) transformation of the coordinate system given by a_{pq}, show that ε_{ijk} satisfies:

$$\varepsilon'_{ijk} = a_{ip} a_{jq} a_{kr} \varepsilon_{pqr}$$

3. If a continuum is subject to a stress S_{ij} at a point P, find expressions for the Normal and total Shear components of force across any plane through P.

4. S is given by

$$S = \begin{bmatrix} S_1 & 0 & 0 \\ 0 & S_2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Find the Normal N and total Shear force S components across a plane with normal
$\underline{\mathbf{n}}^T = (\cos\theta, \sin\theta, 0)$.

Hence show that the pair of values (N, S) lie on a circle in the N, S plane centred at $\{(S_1 + S_2)/2, 0\}$ with radius $|S_1 - S_2|/2$ (This is called the Mohr Circle). Hence find the magnitudes of the maximum Normal and Shear stresses, and the directions they act in.

5. If **F** is the stress force exerted across a plane P, show that the stress force exerted across any plane that contains **F** lies in the plane of P.