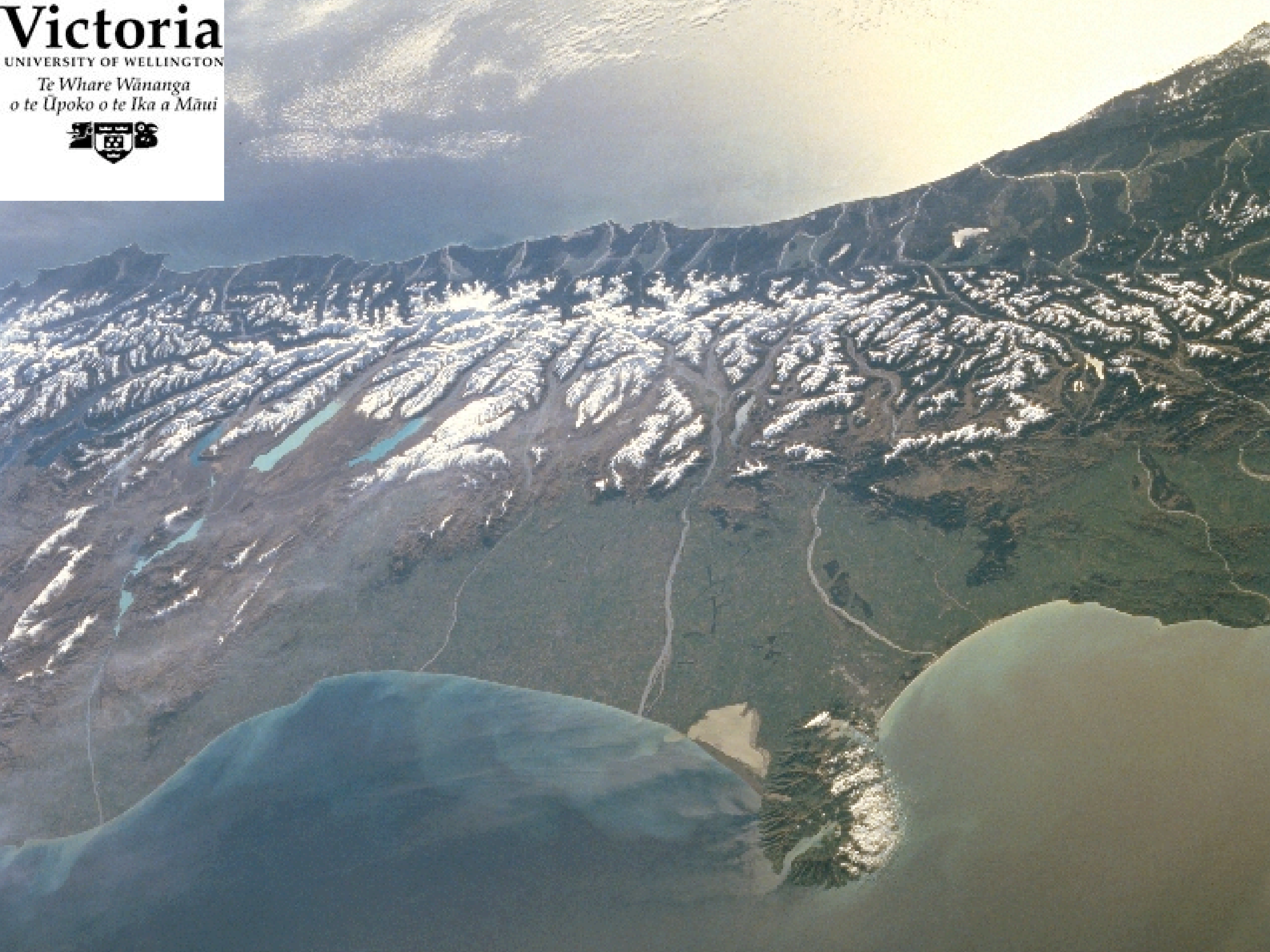
Victoria
UNIVERSITY OF WELLINGTON
Te Whare Wānanga
o te Ūpoko o te Ika a Māui



Victoria
UNIVERSITY OF WELLINGTON
Te Whare Wānanga o te Ūpoko o te Ika a Māui

School of Mathematical and Computing Sciences
Te Kura Pangarau, Rorohiko

Jerk, snap, cosmology

Matt Visser

Abstract:

How much of modern cosmology is really cosmography?

How much of it is kinematics, rather than dynamics?

How much of it is simply FRW symmetries?

How much of it is independent of the Einstein equations?

Symmetry principles give us an awful lot...

Hubble law to high order in redshift...

Precision cosmology?

Cosmography:

Victoria
UNIVERSITY OF WELLINGTON
Te Whare Wānanga
o te Ūpoko o te Ika a Māui

Cosmography uses symmetries to derive the FRW form of the metric:

$$\mathrm{d}s^2 = -c^2\,\mathrm{d}t^2 + a(t)^2\left[\frac{\mathrm{d}r^2}{1-k\,r^2} + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2\right)\right].$$

Cosmography then very pointedly does not invoke the Einstein equations (Friedmann equation).

You can still do an awful lot.

Simply view the history of the scale factor $a(t)$, as a free quantity to be observationally determined.

Cosmography:

After you have observationally determined the complete history of the scale factor, $a(t)$, then you begin to think about dynamics, not before.

Defer all questions of dark matter, dark energy, quintessence, phantom matter, till after you have a good handle on $a(t)$.

Practical issue: You will have to be satisfied with a finite number of derivatives of the scale factor.

Traditionally, the Hubble parameter and the deceleration parameter.

Hubble, deceleration, jerk, snap...

Victoria
UNIVERSITY OF WELLINGTON
Te Whare Wānanga
o te Ūpoko o te Ika a Māui

$$H(t) = +\frac{1}{a}\,\frac{\mathrm{d}a}{\mathrm{d}t}; \qquad q(t) = -\frac{1}{a}\,\frac{\mathrm{d}^2 a}{\mathrm{d}t^2}\,\left[\frac{1}{a}\,\frac{\mathrm{d}a}{\mathrm{d}t}\right]^{-2};$$

$$j(t) = +\frac{1}{a}\,\frac{\mathrm{d}^3 a}{\mathrm{d}t^3}\,\left[\frac{1}{a}\,\frac{\mathrm{d}a}{\mathrm{d}t}\right]^{-3}; \qquad s(t) = +\frac{1}{a}\,\frac{\mathrm{d}^4 a}{\mathrm{d}t^4}\,\left[\frac{1}{a}\,\frac{\mathrm{d}a}{\mathrm{d}t}\right]^{-4}.$$

crackle, pop...

Taylor expand the scale factor using present epoch values:

$$a(t) = a_0 \left\{ 1 + H_0\,(t - t_0) - \frac{1}{2}\,q_0\,H_0^2\,(t-t_0)^2 + \frac{1}{3!}\,j_0\,H_0^3\,(t-t_0)^3 + \frac{1}{4!}\,s_0\,H_0^4\,(t-t_0)^4 + O([t-t_0]^5) \right\}.$$

Hubble law:

Physical distance travelled:

$$D = c \int \mathrm{d}t = c\,(t_0 - t_*),$$

Lookback time: $\Delta t = t_0 - t_*$

Exact (useless) Hubble law:

$$1 + z = \frac{a(t_0)}{a(t_*)} = \frac{a(t_0)}{a(t_0 - \Delta t)} = \frac{a(t_0)}{a(t_0 - D/c)},$$

The useful Hubble law is obtained by Taylor series...

Victoria
UNIVERSITY OF WELLINGTON
Te Whare Wānanga
o te Ūpoko o te Ika a Māui

Hubble law:

$$\frac{a(t_0)}{a(t_0 - D/c)} = 1 + \frac{H_0 D}{c} + \frac{2+q_0}{2}\,\frac{H_0^2 D^2}{c^2} + \frac{6(1+q_0)+j_0}{6}\,\frac{H_0^3 D^3}{c^3} + \frac{24 - s_0 + 8j_0 + 36q_0 + 6q_0^2}{24}\,\frac{H_0^4 D^4}{c^4} + O\left[\left(\frac{H_0 D}{c}\right)^5\right].$$

Redshift:

$$z(D) = \frac{H_0 D}{c} + \frac{2+q_0}{2}\,\frac{H_0^2 D^2}{c^2} + \frac{6(1+q_0)+j_0}{6}\,\frac{H_0^3 D^3}{c^3} + \frac{24 - s_0 + 8j_0 + 36q_0 + 6q_0^2}{24}\,\frac{H_0^4 D^4}{c^4} + O\left[\left(\frac{H_0 D}{c}\right)^5\right].$$

Hubble law:

Victoria
UNIVERSITY OF WELLINGTON
Te Whare Wānanga
o te Ūpoko o te Ika a Māui

Reversion of series:

$$D(z) = \frac{c\, z}{H_0}\left\{1 - \left[1+\frac{q_0}{2}\right] z + \left[1 + q_0 + \frac{q_0^2}{2} - \frac{j_0}{6}\right] z^2 \right.$$
$$\left. - \left[1 + \frac{3}{2} q_0 (1+q_0) + \frac{5}{8} q_0^3 - \frac{1}{2} j_0 - \frac{5}{12} q_0 j_0 - \frac{s_0}{24}\right] z^3 + O(z^4)\right\}.$$

Usual Hubble law in terms of luminosity distance:

$$(\text{energy flux}) = \frac{L}{4\pi\, d_L^2}.$$

Pure geometry:

$$d_L = a(t_0)^2\, \frac{r_0}{a(t_*)} = \frac{a_0}{a(t_0 - D/c)}\, (a_0\, r_0).$$

Hubble law (technical mess):

Victoria
UNIVERSITY OF WELLINGTON
Te Whare Wānanga
o te Ūpoko o te Ika a Māui

$$r_0(D) = \begin{cases} \sin\left(\int_{t_0-D/c}^{t_0} \frac{c\,\mathrm{d}t}{a(t)}\right) & k=+1; \\ \int_{t_0-D/c}^{t_0} \frac{c\,\mathrm{d}t}{a(t)} & k=0; \\ \sinh\left(\int_{t_0-D/c}^{t_0} \frac{c\,\mathrm{d}t}{a(t)}\right) & k=-1; \end{cases}$$

$$r_0(D) = \left[\int_{t_0-D/c}^{t_0} \frac{c\,\mathrm{d}t}{a(t)}\right] - \frac{k}{3!}\left[\int_{t_0-D/c}^{t_0} \frac{c\,\mathrm{d}t}{a(t)}\right]^3 + O\left(\left[\int_{t_0-D/c}^{t_0} \frac{c\,\mathrm{d}t}{a(t)}\right]^5\right),$$

$$\int_{t_0-D/c}^{t_0} \frac{c\,\mathrm{d}t}{a(t)} = \frac{D}{a_0}\left\{1 + \frac{1}{2}\frac{H_0 D}{c} + \left[\frac{2+q_0}{6}\right]\left(\frac{H_0 D}{c}\right)^2 + \left[\frac{6(1+q_0)+j_0}{24}\right]\left(\frac{H_0 D}{c}\right)^3 + O\left[\left(\frac{H_0 D}{c}\right)^4\right]\right\}.$$

Insert, collect terms...

Hubble law (technical mess):

Victoria University of Wellington
Te Whare Wānanga o te Ūpoko o te Ika a Māui

$$r_0(D) = \frac{D}{a_0}\left\{1 + \frac{1}{2}\frac{H_0 D}{c} + \frac{1}{6}\left[2 + q_0 - \frac{kc^2}{H_0^2 a_0^2}\right]\left(\frac{H_0 D}{c}\right)^2 + \frac{1}{24}\left[6(1+q_0) + j_0 - 6\frac{kc^2}{H_0^2 a_0^2}\right]\left(\frac{H_0 D}{c}\right)^3 + O\left[\left(\frac{H_0 D}{c}\right)^4\right]\right\}.$$

$$d_L(D) = D\left\{1 + \frac{3}{2}\left(\frac{H_0 D}{c}\right) + \frac{1}{6}\left[11 + 4q_0 - \frac{kc^2}{H_0^2 a_0^2}\right]\left(\frac{H_0 D}{c}\right)^2 + \frac{1}{24}\left[50 + 40q_0 + 5j_0 - 10\frac{kc^2}{H_0^2 a_0^2}\right]\left(\frac{H_0 D}{c}\right)^3 + O\left[\left(\frac{H_0 D}{c}\right)^4\right]\right\}.$$

$$d_L(z) = \frac{c\,z}{H_0}\left\{1 + \frac{1}{2}\left[1 - q_0\right] z - \frac{1}{6}\left[1 - q_0 - 3q_0^2 + j_0 + \frac{kc^2}{H_0^2 a_0^2}\right] z^2 + \frac{1}{24}\left[2 - 2q_0 - 15q_0^2 - 15q_0^3 + 5j_0 + 10q_0 j_0 + s_0 + \frac{2kc^2(1+3q_0)}{H_0^2 a_0^2}\right] z^3 + O(z^4)\right\}.$$

Hubble law (luminosity distance):

Victoria University of Wellington
Te Whare Wānanga o te Ūpoko o te Ika a Māui

$$d_L(z) = \frac{c\,z}{H_0}\left\{1 + \frac{1}{2}\left[1 - q_0\right] z - \frac{1}{6}\left[1 - q_0 - 3q_0^2 + j_0 + \frac{kc^2}{H_0^2 a_0^2}\right] z^2 \right.$$
$$+ \frac{1}{24}\left[2 - 2q_0 - 15q_0^2 - 15q_0^3 + 5j_0 + 10q_0 j_0 + s_0 + \frac{2kc^2(1+3q_0)}{H_0^2 a_0^2}\right] z^3$$
$$\left. + O(z^4) \right\}.$$

The first two terms are standard.
The third term is that of Chiba & Nakamura.
The fourth term is new.
With Maple, can calculate to arbitrary order.

Third order (jerk) begins to probe geometry of space.
More free parameters than coefficients to fit...

Hubble law (assuming cosmological inflation):

Victoria
UNIVERSITY OF WELLINGTON
Te Whare Wānanga
o te Ūpoko o te Ika a Māui

$H_0\, a_0/c \gg 1$ Not $k = 0.$

$$d_L(z) = \frac{c\, z}{H_0}\left\{1 + \frac{1}{2}\left[1 - q_0\right] z - \frac{1}{6}\left[1 - q_0 - 3q_0^2 + j_0\right] z^2 + \frac{1}{24}\left[2 - 2q_0 - 15q_0^2 - 15q_0^3 + 5j_0 + 10q_0 j_0 + s_0\right] z^3 + O(z^4)\right\}.$$

Number of free parameters now equals number of coefficients ...

If and only if you have independent means (observational or theoretical) for bounding $H_0\, a_0/c$ can you even hope to constrain jerk and snap from the observational Hubble law.

Completely independent of the Einstein equations.

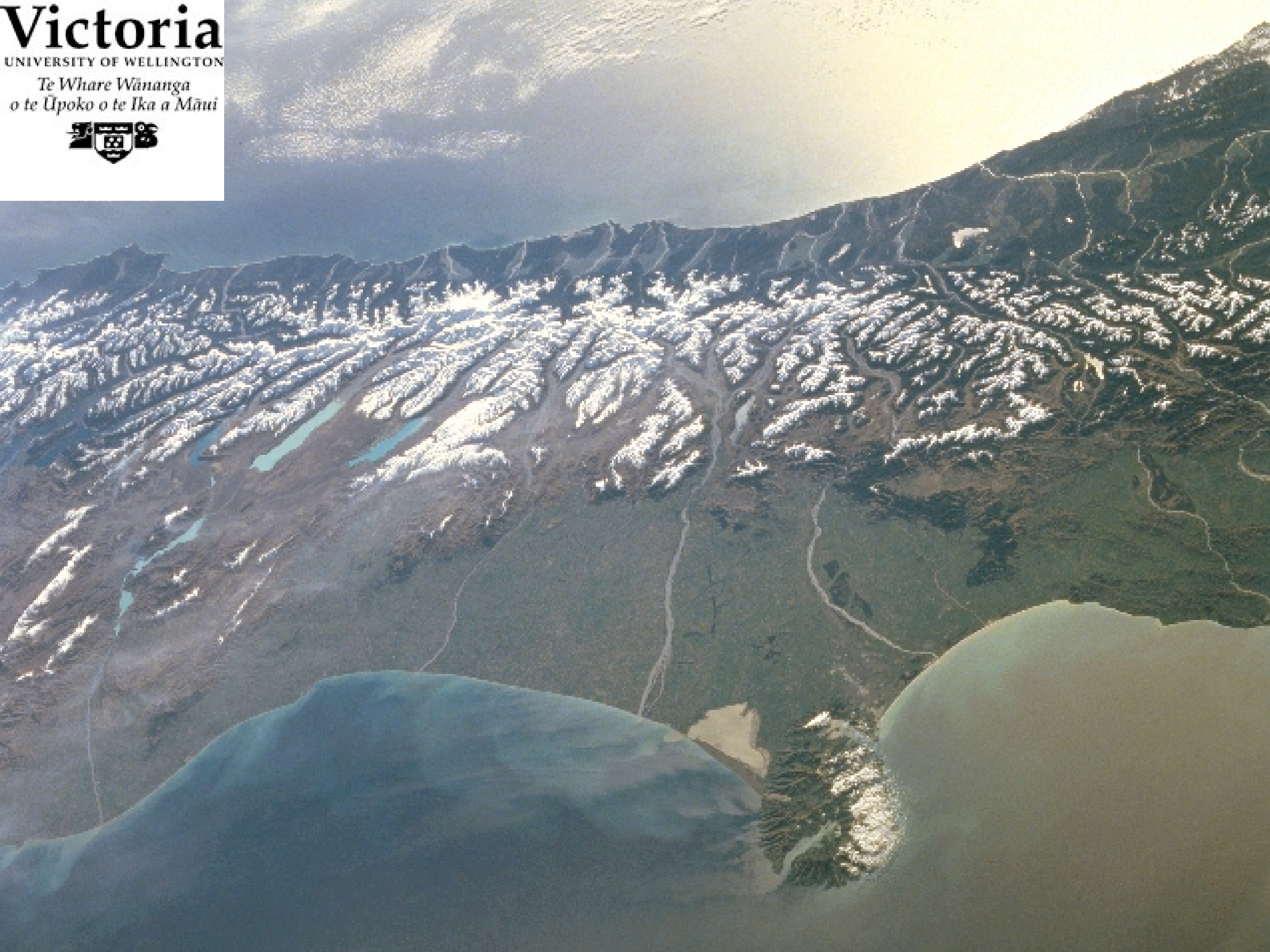
Conclusions:

Cosmography can teach us an awful lot.

Even without the Einstein equations,
symmetry and FRW cosmology
gives you the Hubble law.

The Einstein equations come in when you then
try to relate the Hubble law back to the
cosmological matter model.

Victoria
UNIVERSITY OF WELLINGTON
Te Whare Wānanga
o te Ūpoko o te Ika a Māui



Cosmological fluid:

$$p = p_0 + \kappa_0\,(\rho - \rho_0) + \frac{1}{2}\,\frac{\mathrm{d}^2 p}{\mathrm{d}\rho^2}\bigg|_0 (\rho - \rho_0)^2 + O[(\rho - \rho_0)^3],$$

Friedmann equations:

$$8\pi G_N\,\rho(t) = 3c^2\left[\frac{\dot{a}^2}{a^2} + \frac{kc^2}{a^2}\right];$$

$$8\pi G_N\,p(t) = -c^2\left[\frac{\dot{a}^2}{a^2} + \frac{kc^2}{a^2} + 2\frac{\ddot{a}}{a}\right].$$

w-parameter:

$$w(t) = \frac{p}{\rho} = -\frac{H^2(1-2q) + kc^2/a^2}{3(H^2 + kc^2/a^2)} = -\frac{(1-2q) + kc^2/(H^2a^2)}{3[1 + kc^2/(H^2a^2)]}.$$

Victoria
UNIVERSITY OF WELLINGTON
Te Whare Wānanga
o te Ūpoko o te Ika a Māui

Cosmological fluid:

Victoria University of Wellington
Te Whare Wānanga o te Ūpoko o te Ika a Māui

$$\kappa_0 = \left.\frac{\mathrm{d}p}{\mathrm{d}\rho}\right|_0 .$$

$$\left\{ \begin{aligned} 8\pi G_N\, \rho(t) &= 3c^2 \left[\frac{\dot{a}^2}{a^2} + \frac{kc^2}{a^2}\right]; \\ 8\pi G_N\, p(t) &= -c^2 \left[\frac{\dot{a}^2}{a^2} + \frac{kc^2}{a^2} + 2\frac{\ddot{a}}{a}\right]. \end{aligned} \right.$$

$$\kappa_0 = -\frac{1}{3} \left[\frac{1 - j_0 + kc^2/(H_0^2 a_0^2)}{1 + q_0 + kc^2/(H_0^2 a_0^2)}\right],$$

$$\kappa_0 = -\frac{1}{3} \left[\frac{1 - j_0}{1 + q_0}\right].$$

(Cosmological inflation)

Determining κ_0 requires a measurement of jerk.

Cosmological fluid:

Victoria
UNIVERSITY OF WELLINGTON
Te Whare Wānanga
o te Ūpoko o te Ika a Māui

$$\left.\frac{\mathrm{d}^2 p}{\mathrm{d}\rho^2}\right|_0 = -\frac{(1+kc^2/[H_0^2 a_0^2])}{6\rho_0(1+q_0+kc^2/[H_0^2 a_0^2])^3}\Big\{s_0(1+q_0)+j_0(1+j_0+4q_0+q_0^2)+q_0(1+2q_0)$$
$$+(s_0+j_0+q_0+q_0 j_0)\frac{kc^2}{H_0^2 a_0^2}\Big\}.$$

Assuming cosmological inflation:

$$\left.\frac{\mathrm{d}^2 p}{\mathrm{d}\rho^2}\right|_0 = -\frac{s_0(1+q_0)+j_0(1+j_0+4q_0+q_0^2)+q_0(1+2q_0)}{6\rho_0(1+q_0)^3}.$$

Note, need both jerk and snap.

This is what makes cosmology so difficult;
low-order coefficients in the EOS need
high-order coefficients in Hubble.

Victoria
UNIVERSITY OF WELLINGTON
Te Whare Wānanga o te Ūpoko o te Ika a Māui


Mt. Taranaki
Egmont National Park
Boundary