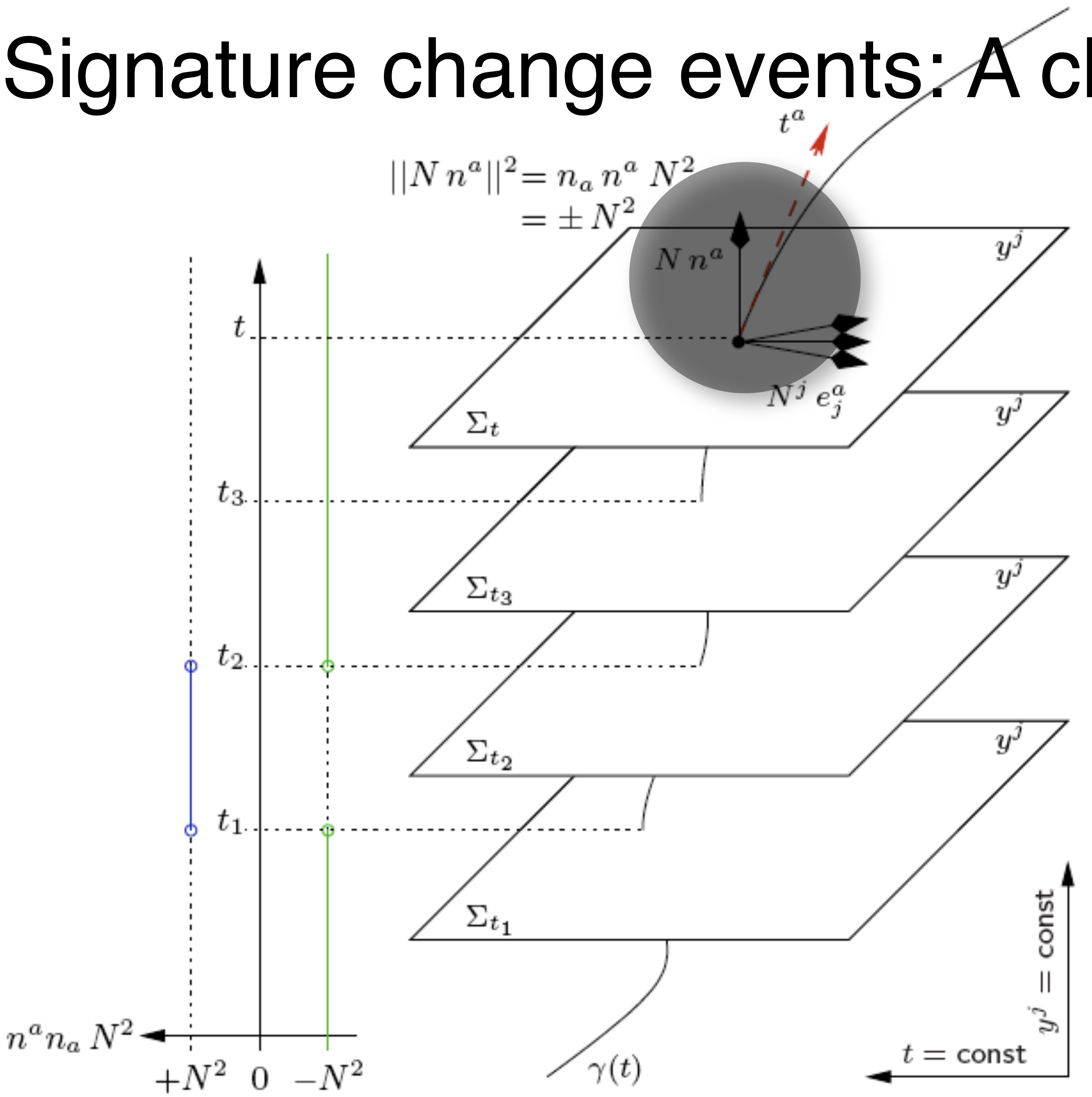
Signature change events: A challenge for quantum gravity?



By
Silke Weinfurtner
in collaboration with
Angela White
Matt Visser

THE UNIVERSITY OF BRITISH COLUMBIA | VANCOUVER

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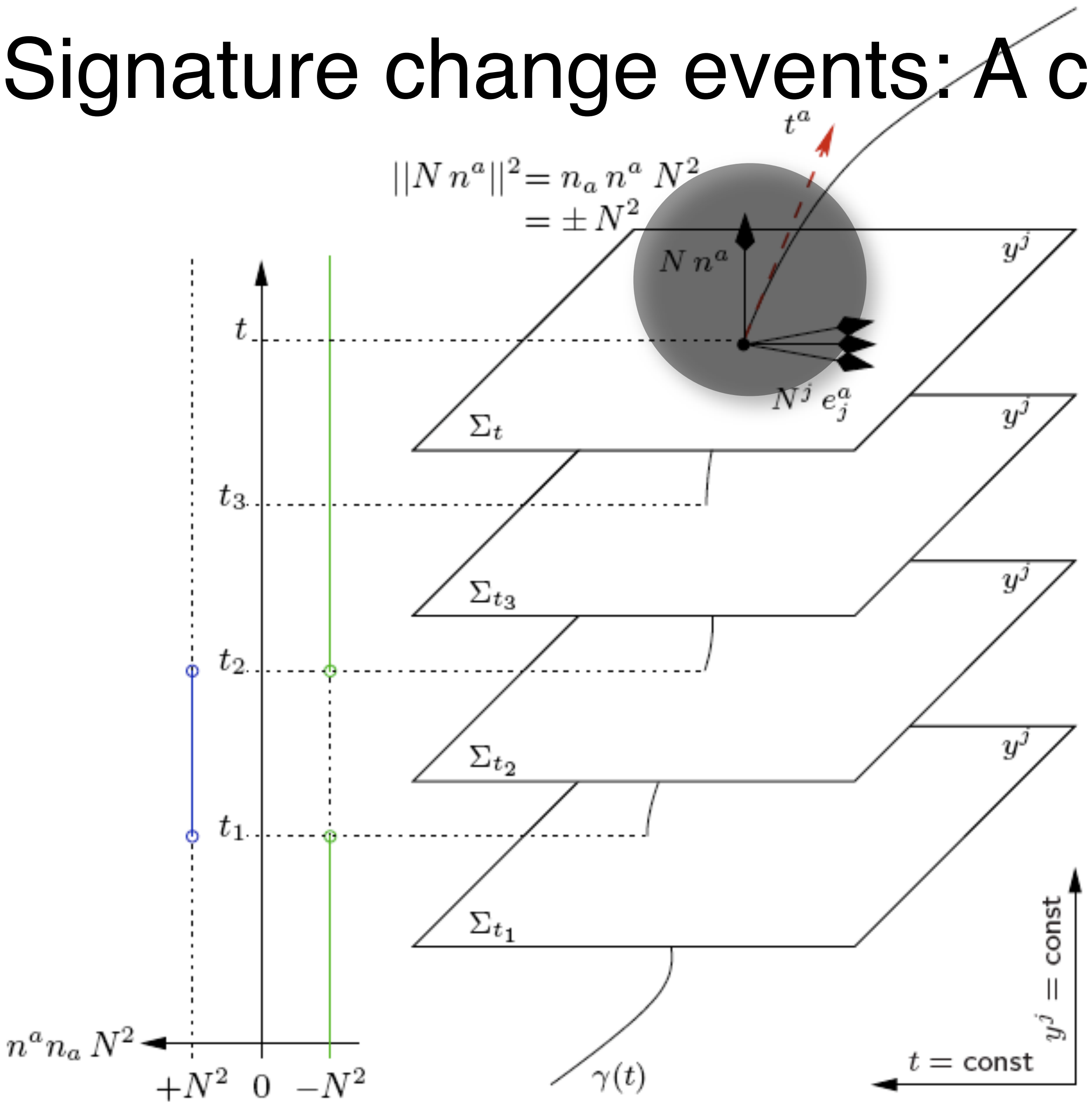

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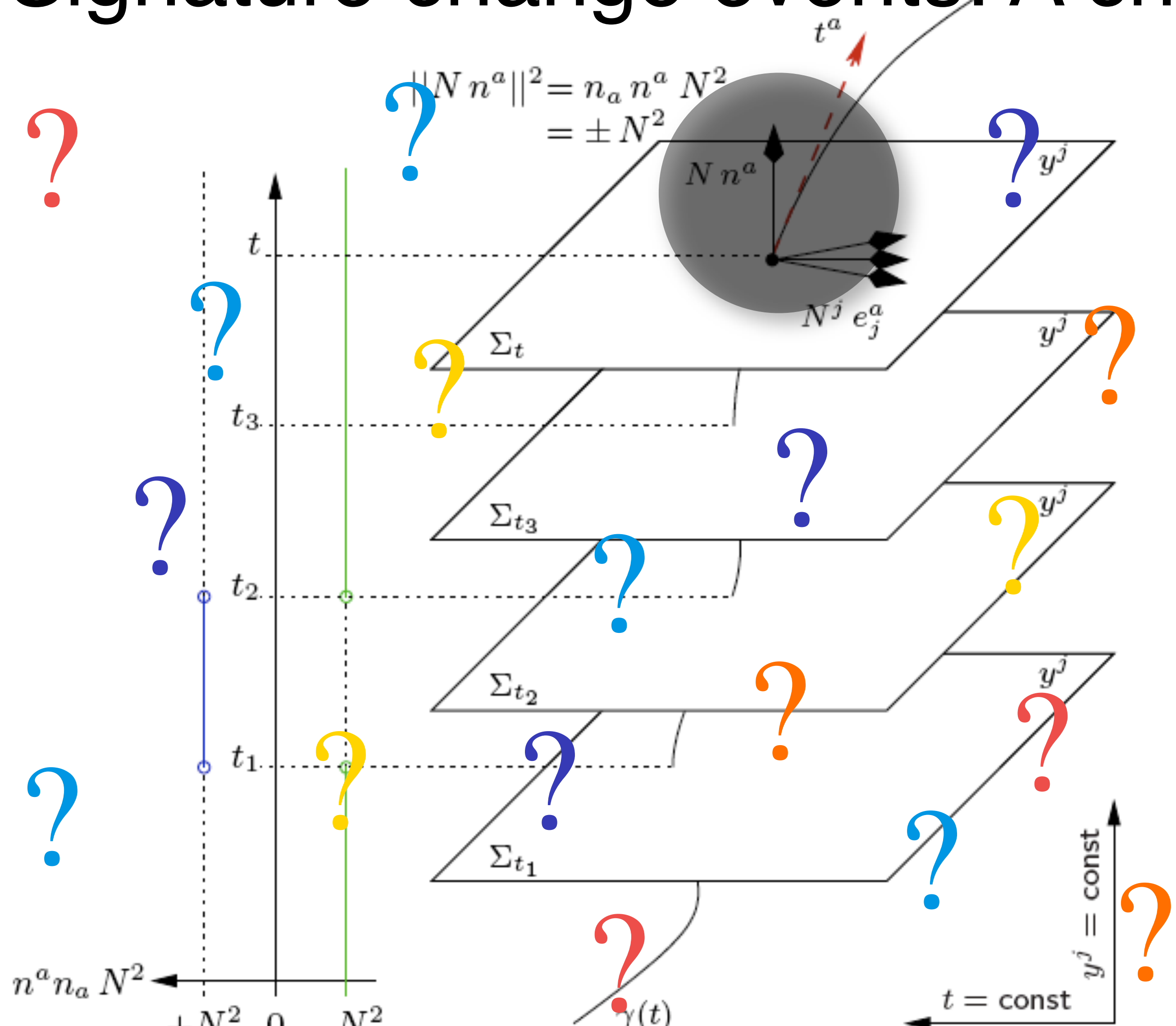

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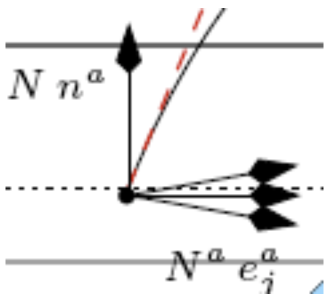
Outlook

Motivation from Emergent spacetimes



Signature of spacetime - where does it come from?

Quantum field theory on Riemannian manifolds

Trans-Planckian beats signature?

What, **if** anything at all, did we learn from this!?

A really really long...

- Motivation from Emergent spacetimes

The concept of emergence

Emergent spacetimes involve…

- A microscopic system of fundamental objects (e.g. strings, atoms or molecules);
- a dominant mean field regime, where the microscopic degrees of freedom give way to collective variables;
- a geometrical object (e.g. a symmetric tensor dominating the evolution of linearized classical and quantum excitations around the mean field;
- An emergent Lorentz symmetry for the long-distance behavior of the geometrical object;

Example BEC [microscopic degrees of freedom]

Emergent spacetimes from Bose-gas

- A microscopic system of fundamental objects:
  ultra-cold dilute gas of weakly interacting Bosons

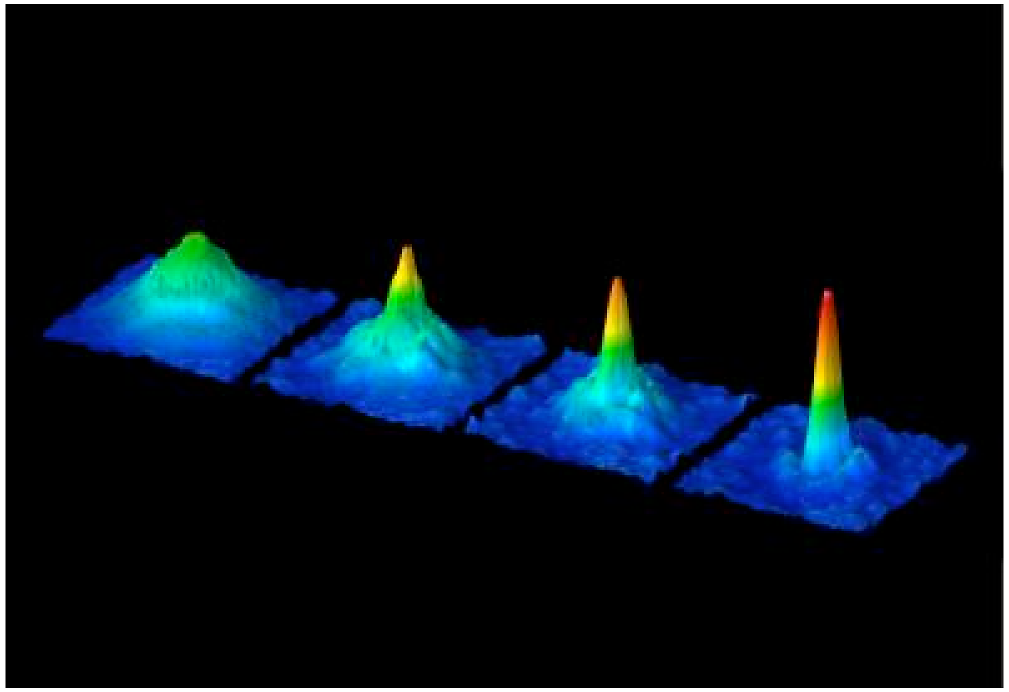
Microscopic theory well understood:

$$\hat{H} = \int d\mathbf{x} \left(-\hat{\Psi}^\dagger \frac{\hbar^2}{2m} \nabla^2 \hat{\Psi} + \hat{\Psi}^\dagger V_{\mathbf{ext}} \hat{\Psi} + \frac{U}{2} \hat{\Psi}^\dagger \hat{\Psi}^\dagger \hat{\Psi} \hat{\Psi} \right)$$

$\mathrm{SO(2) - symmetry}$ $\quad \hat{\Psi} \to \hat{\Psi}^* = \hat{\Psi} \exp(i\alpha)$

Example BEC [macroscopic variables]

Emergent spacetimes from Bose-gas



- A dominant mean field regime:

  Bose-Einstein condensate

Spontaneous symmetry breaking:

$$\langle \hat{\Psi}(t, \mathbf{x}) \rangle = \psi(t, \mathbf{x}) = \sqrt{n_0(t, \mathbf{x})} \exp(i\phi_0(t, \mathbf{x})) \neq 0$$

Example BEC [geometrical object]

Emergent spacetimes from Bose-gas

Small perturbations - linear in density and phase - in the macroscopic mean-field emerging from an ultra-cold weakly interacting gas of bosons are inner observers experiencing an effective spacetime geometry,

$$\frac{1}{\sqrt{|\det(g_{ab})|}}\,\partial_a\left(\sqrt{|\det(g_{ab})|}\;g^{ab}\;\partial_b\hat{\phi}\right)=0$$

where

$$g_{ab}=\left(\frac{c_0}{U/\hbar}\right)^{\frac{2}{d-1}}\begin{bmatrix} -\left(c_0^2-v^2\right) & -v_x & -v_y & -v_z \\ -v_x & 1 & 0 & 0 \\ -v_y & 0 & 1 & 0 \\ -v_z & 0 & 0 & 1 \end{bmatrix};$$

Semi-classical quantum geometry

C. Barcelo, S. Liberati, and M. Visser. Analog gravity from field theory normal modes? *Class. Quant. Grav.*, 18:3595–3610, 2001.

Effective curved-spacetime quantum field theory description of the linearization process:

Small perturbations around some background solution $\phi_0(t,\mathbf{x})$

$$\phi(t,\mathbf{x}) = \phi_0(t,\mathbf{x}) + \epsilon\,\phi_1(t,\mathbf{x}) + \frac{\epsilon^2}{2}\,\phi_2(t,\mathbf{x})$$

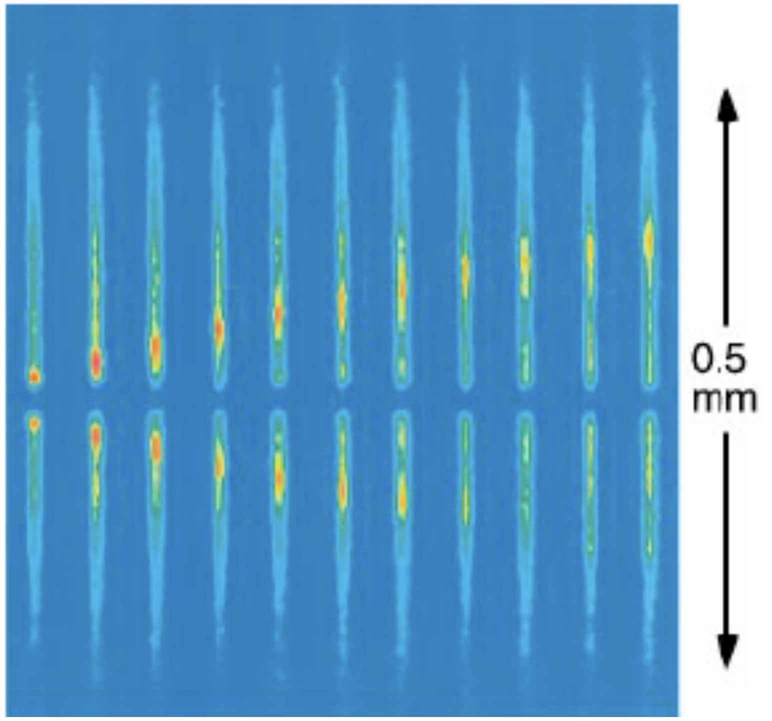
In a generic Lagrangian $\mathcal{L}(\partial_a\phi,\phi)$, depdending only a single Scalar field and its first derivatives yields an effective Spacetime geometry

$$g_{ab}(\phi_0) = \left[-\det\left(\frac{\partial^2\mathcal{L}}{\partial(\partial_a\phi)\,\partial(\partial_b\phi)}\right)\right]^{\frac{1}{d-1}}\Bigg|_{\phi_0}\left(\frac{\partial^2\mathcal{L}}{\partial(\partial_a\phi)\,\partial(\partial_b\phi)}\right)^{-1}\Bigg|_{\phi_0}$$

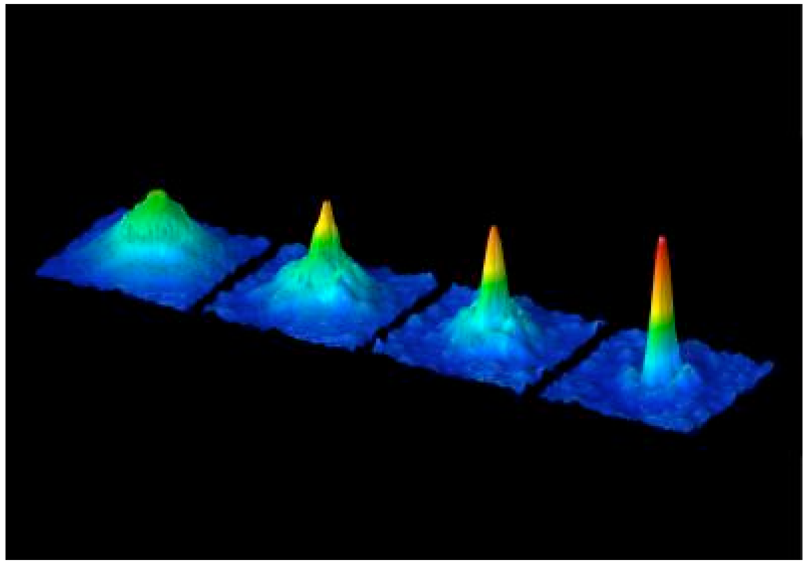
For the classical/ quantum fluctuations. The equation of Motion for small perturbations around the background Are then given by $\left(\Delta_{g(\phi_0)} - V(\phi_0)\right)\phi_1 = 0$

Kinematics versus dynamics!

Inner and out observer/ absolute time



Inner observer:
Small excitations in the system experience an effective spacetime geometry represented by the macroscopic mean-field variables!



Outer observer:
Live in the preferred frame - the laboratory frame, such that the condensate parameters are functions of lab-time (absolute time).

Interactions and spacetime signature

$$\vec{v} \to \vec{0}$$

$$U \to U(t)$$

$$c_0^2 \to c(t)^2$$

$$g_{ab} = \left(\frac{c(t)}{U(t)/\hbar} \right)^{\frac{2}{d-1}} \begin{bmatrix} -c(t)^2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$c_0^2 = \frac{n_0(t, \mathbf{x})\, U(t)}{m} \qquad \begin{array}{ll} U > 0 & \text{repulsive}\,; \\ U < 0 & \text{attractive}\,. \end{array}$$

Interactions and spacetime signature

$$\vec{v} \to \vec{0}$$

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$$g_{ab} = \left(\frac{c(t)}{U(t)/\hbar} \right)^{\frac{2}{d-1}} \begin{bmatrix} -c(t)^2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$c_0^2 = \frac{n_0(t, \mathbf{x})\, U(t)}{m}$$

$$U > 0 \qquad \text{repulsive}\,;$$
$$U < 0 \qquad \text{attractive}\,.$$

$$g_{ab} \sim \begin{bmatrix} -c(t)^2 & 0 & 0 & 0 \\ 0 & +1 & 0 & 0 \\ 0 & 0 & +1 & 0 \\ 0 & 0 & 0 & +1 \end{bmatrix}$$

Lorentzian signature

$$g_{ab} \sim \begin{bmatrix} +c(t)^2 & 0 & 0 & 0 \\ 0 & +1 & 0 & 0 \\ 0 & 0 & +1 & 0 \\ 0 & 0 & 0 & +1 \end{bmatrix}$$

Riemannian signature

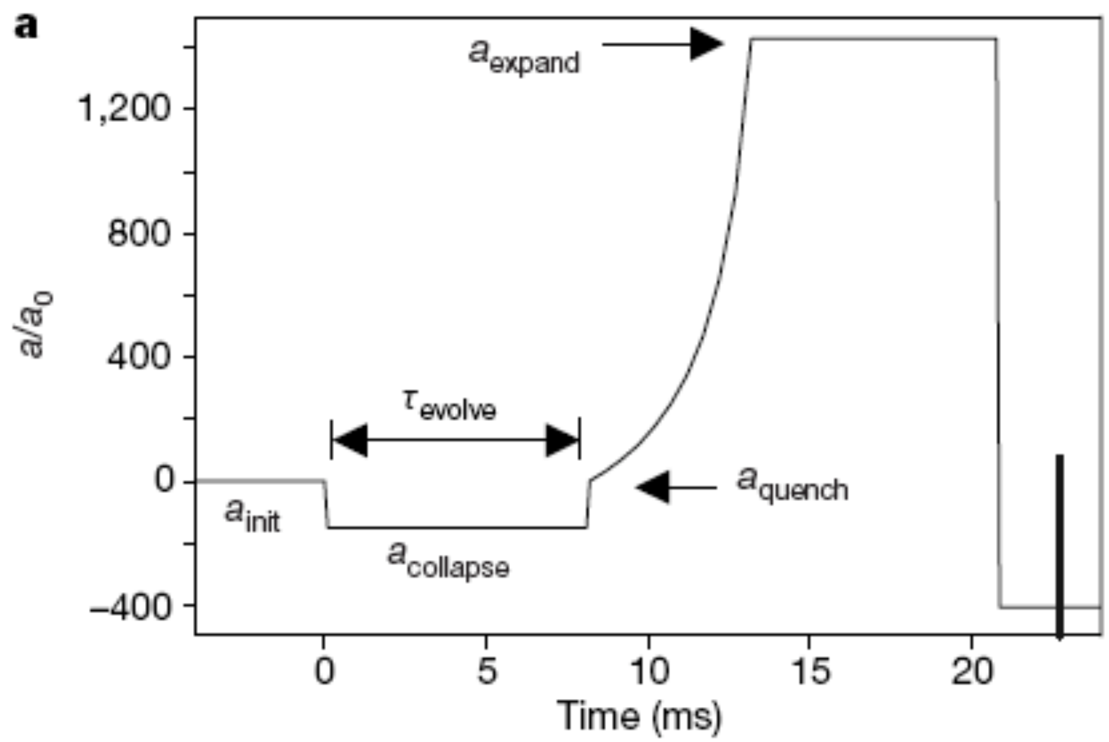
Daily signature change events...

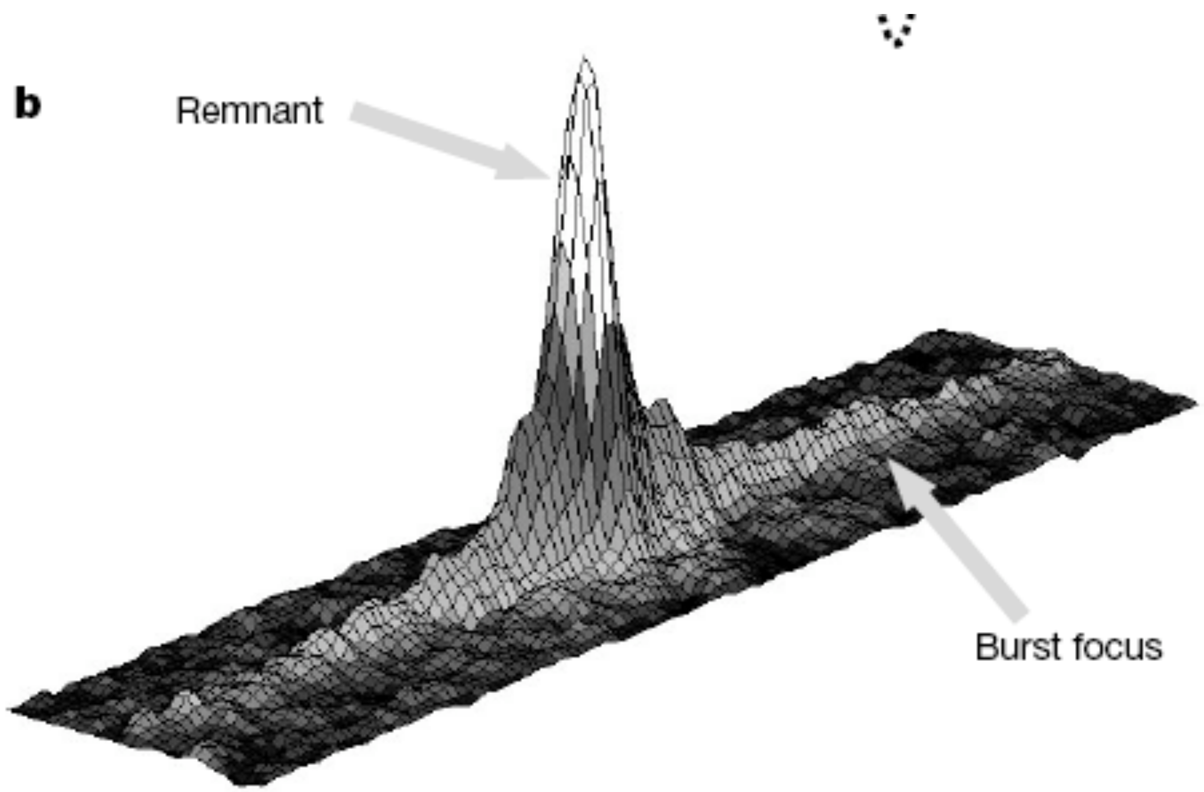
Dynamics of collapsing and exploding Bose–Einstein condensates

NATURE | VOL 412 | 19 JULY 2001

Elizabeth A. Donley*, Neil R. Claussen*, Simon L. Cornish*, Jacob L. Roberts*, Eric A. Cornell*† & Carl E. Wieman*

The bosenova experiment:





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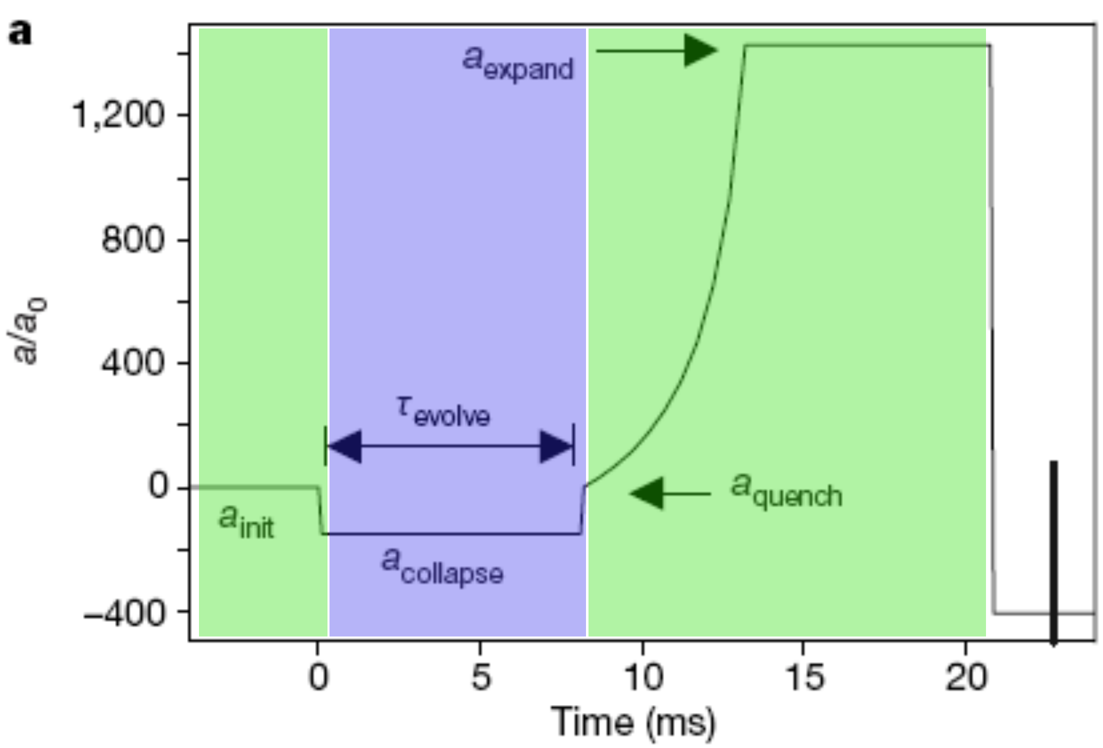
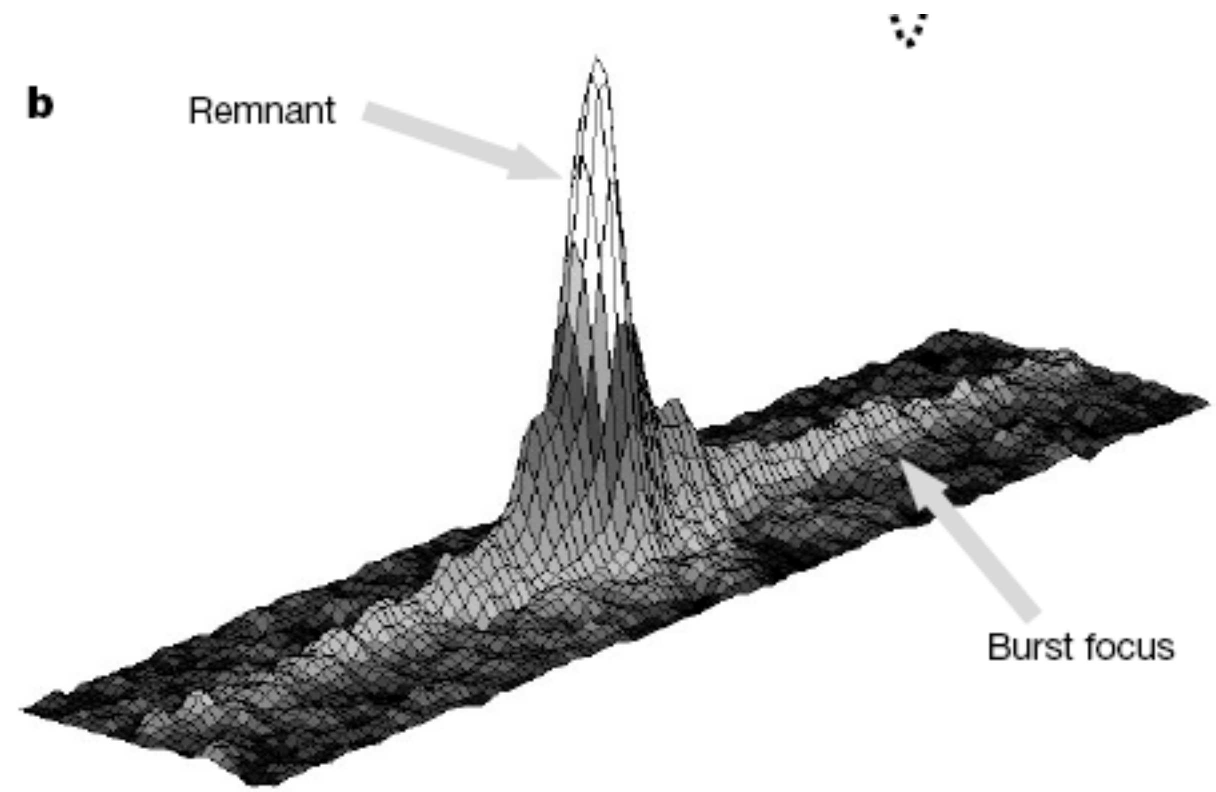
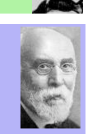




Lorentzian signature 

Riemannian signature 

Daily signature change events...

Can we understand the bosenova experiment via the emergent spacetime programme?

Early Universe Quantum Processes in BEC Collapse Experiments

E. A. Calzetta[1] and B. L. Hu[2] *

[1]Departamento de Fisica, FCEyN Universidad de Buenos Aires Ciudad Universitaria, 1428 Buenos Aires, Argentina

[2]Department of Physics, University of Maryland, College Park, MD 20742, USA

(March 11, 2005)

- *Invited Talk presented at the Peyresq Meetings of Gravitation and Cosmology, 2003. To appear in Int. J. Theor. Phys.*

Main Theme We show that in the collapse of a Bose-Einstein condensate (BEC) [1] certain processes involved and mechanisms at work share a common origin with corresponding quantum field processes in the early universe such as particle creation, structure formation and spinodal instability. Phenomena associated with the controlled BEC collapse observed in the experiment of Donley et al [2] (they call it 'Bose-Nova', see also [3]) such as the appearance of bursts and jets can be explained as a consequence of the squeezing and amplification of quantum fluctuations above the condensate by the dynamics of the condensate. Using the

Need to understand particle production process via sudden variations in atomic-interactions...

Atom-interactions, universe and quasiparticles

Variations in the speed of sound modify the size of emergent universe!

$$c_0^2 = \frac{n_0(t,\mathbf{x})\,U(t)}{m}$$

Outer observer: For an decreasing sound velocity signals.
Inner observer: The size of the universe has increased.

Time-dependent atom-interactions correspond to FRW-type universe

$$ds^2 = \left(\frac{n_0}{c_0}\right)^{\frac{2}{d-1}} \left[-c_0^2\, b_k(t)^\alpha\, dt^2 + b_k(t)^{\alpha-1}\, d\mathbf{x}^2\right]$$

quasiparticle production is expected!!!

Simplest example: Sudden changes in the emergent scale factor.

Atom-interactions, universe and quasiparticles

Variations in the speed of sound modify the size of emergent universe!

$$c_0^2 = \frac{n_0(t,\mathbf{x})\,U(t)}{m}$$

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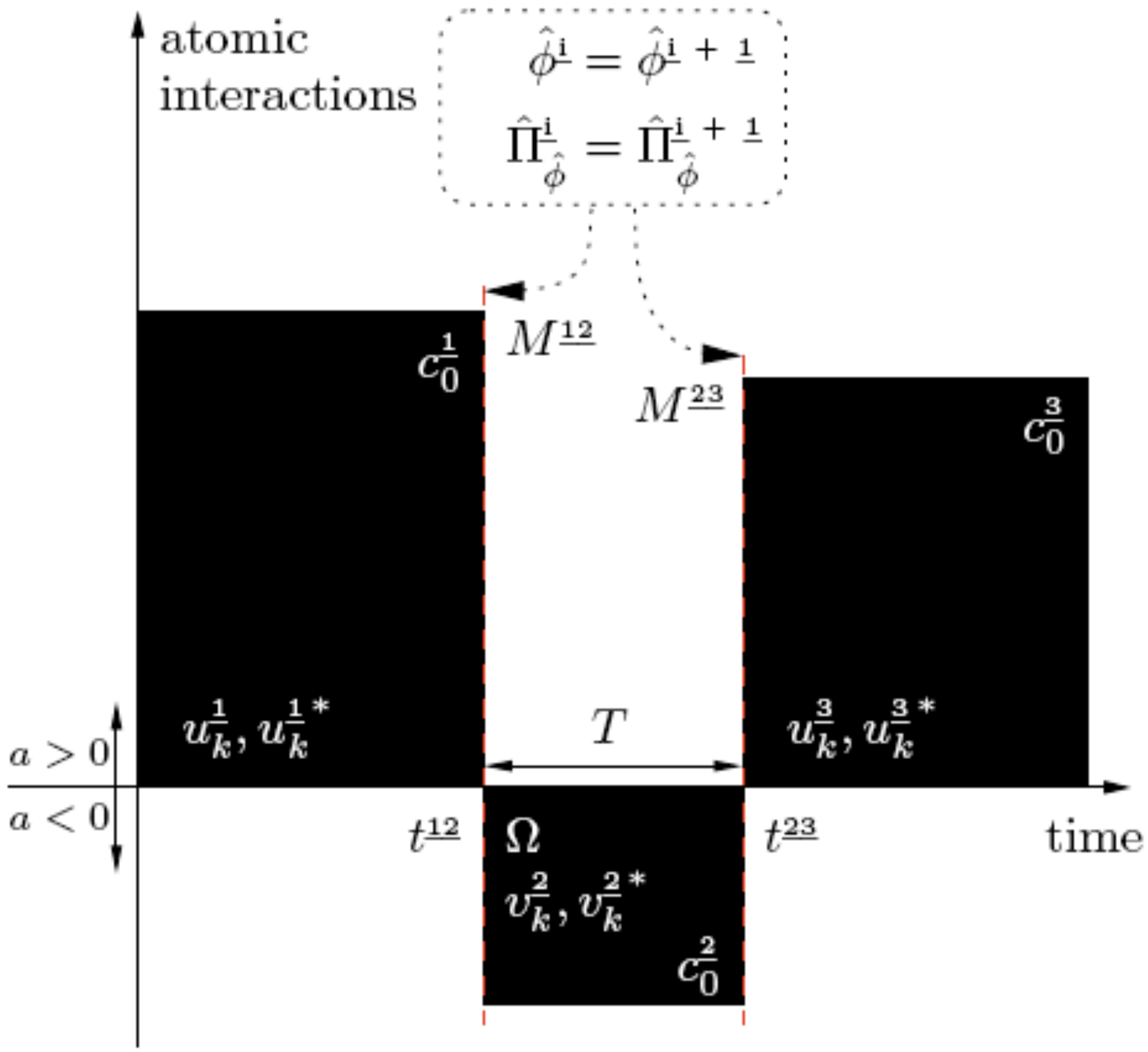
Trans-Planckian beats signature



+ no meaning of quasiparticles in the intermediate regime!!!

+ upper / lower bound

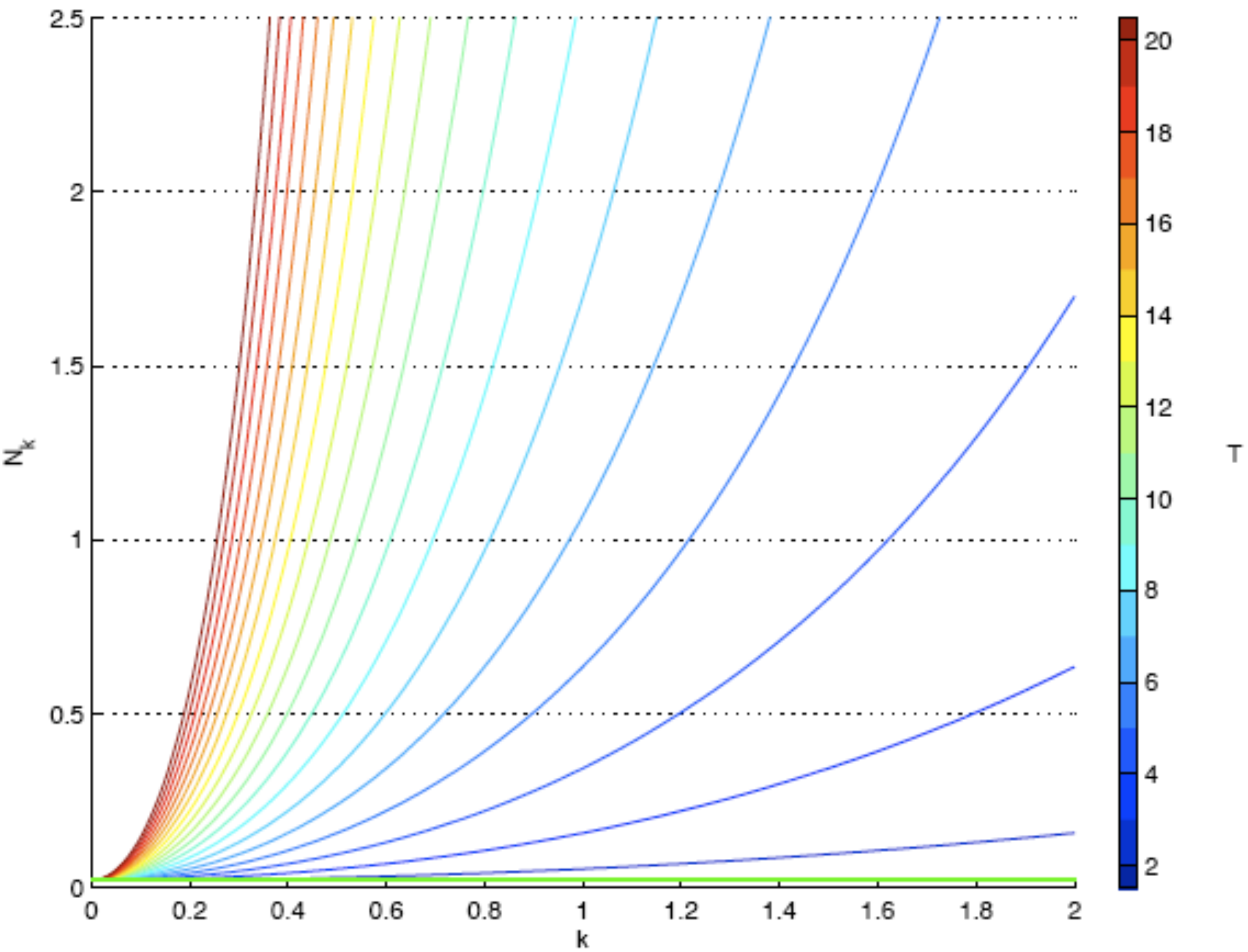
+ quasiparticle production

+ connection conditions

$$\left[\hat{\phi}\right]_{\Sigma} = 0; \qquad \left[\hat{\Pi}_{\hat{\phi}}\right]_{\Sigma} = 0.$$

Trans-Planckian beats signature

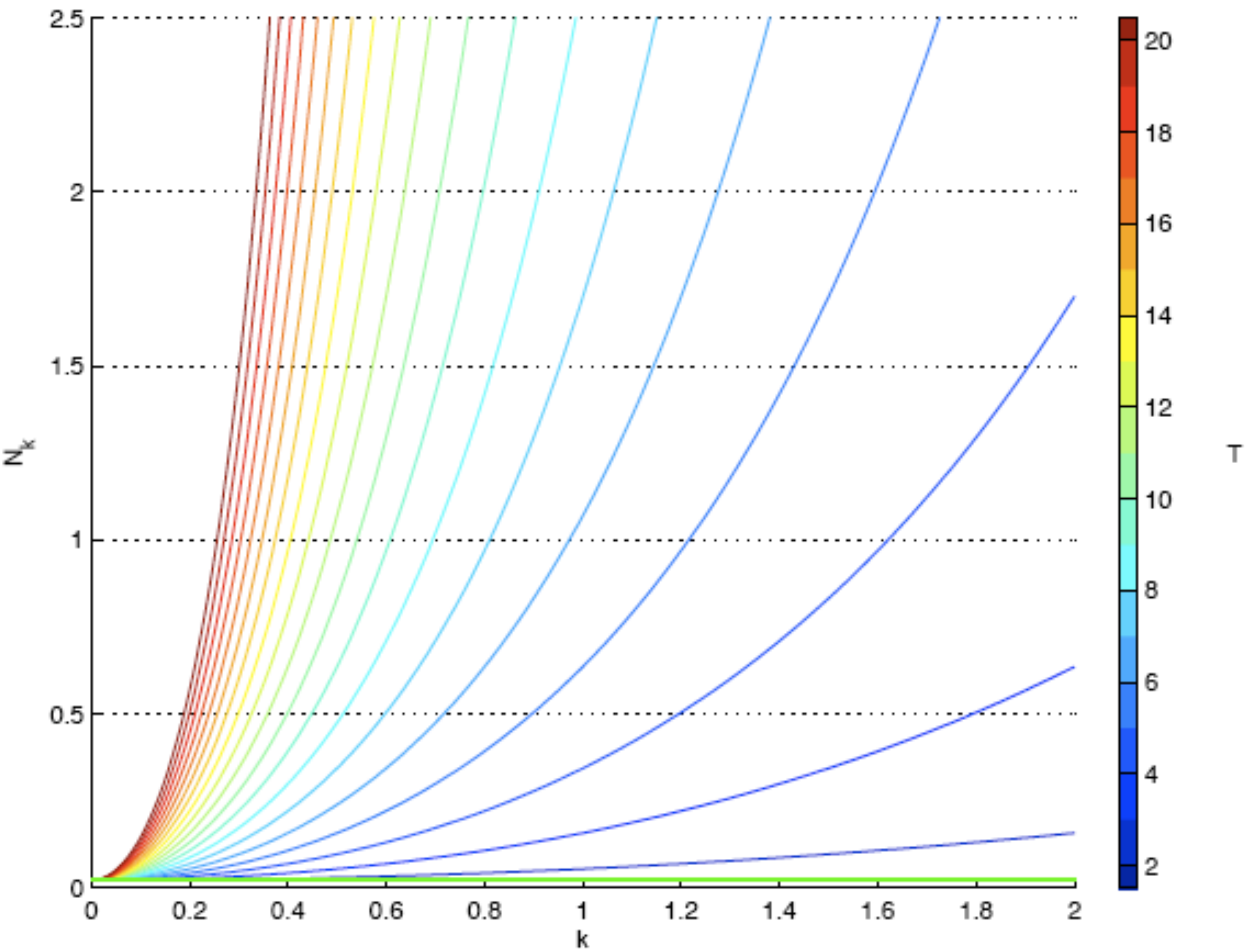
hydrodynamic
approximation



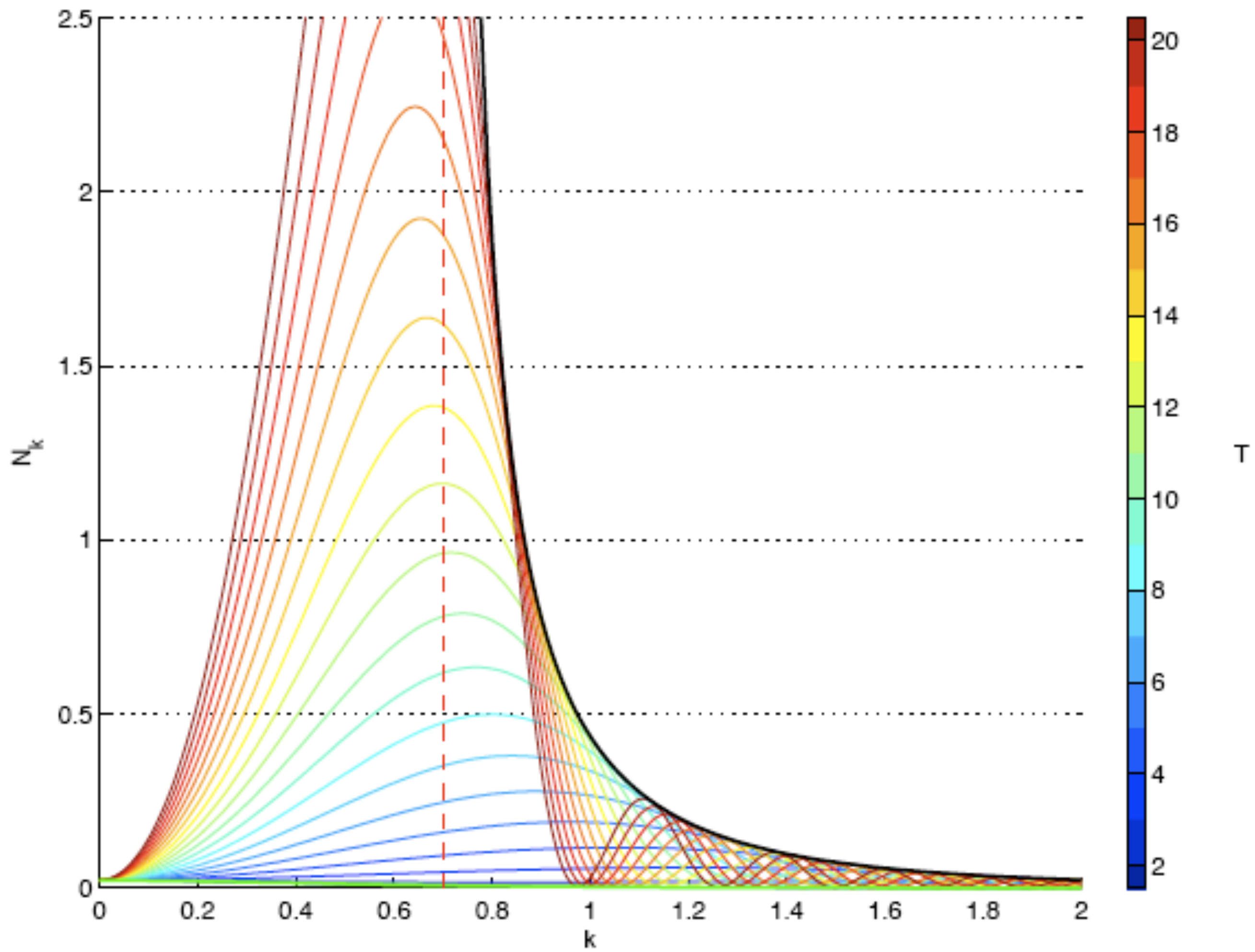
Number of quasiparticles
infinite!?

Trans-Planckian beats signature

hydrodynamic approximation



modified hydrodynamics
[including quantum pressure effects]



Trans-Planckian beats signature

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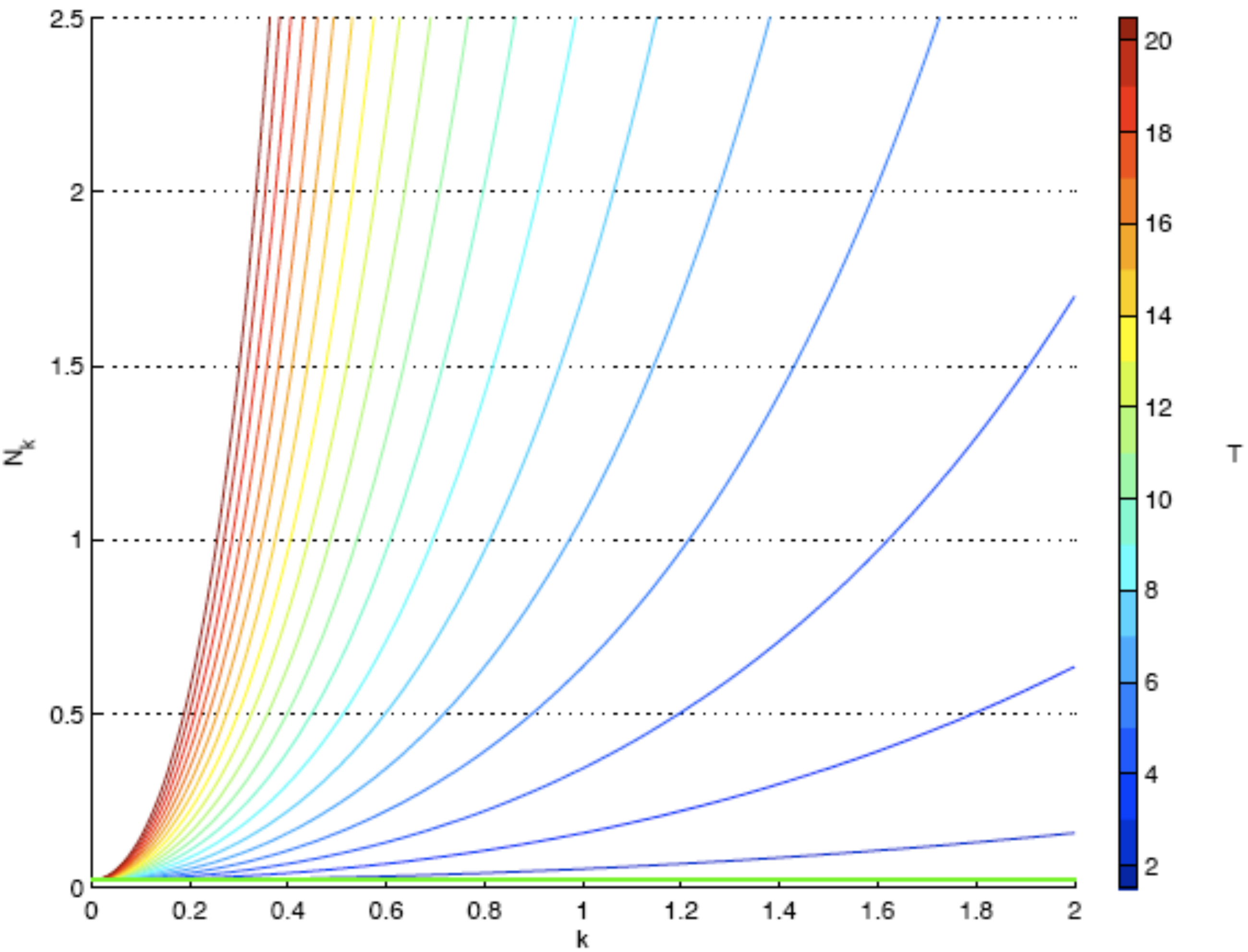

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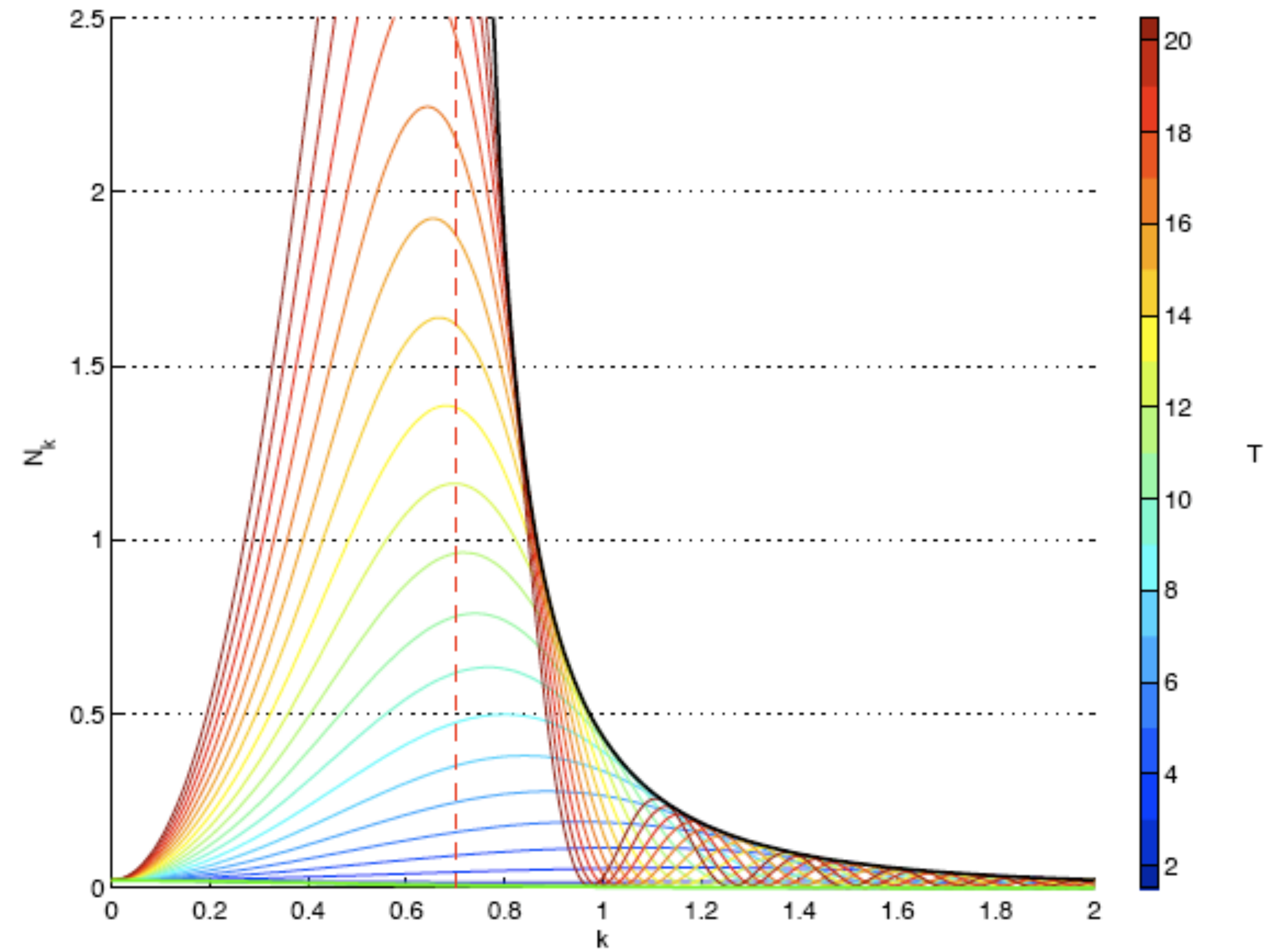


$$\mathcal{U}|_{\nabla \to -ik} \to U_k = U + \frac{\hbar^2}{4mn_0}k^2$$

Trans-Planckian beats signature

Hydrodynamic approximation: Variations in the kinetic energy of the condensate are considered to be negligible, compared to the internal potential energy of the Bosons.

$$\frac{\hbar^2}{2m}\frac{\nabla^2\sqrt{n_0+\hat{n}}}{\sqrt{n_0+\hat{n}}} \ll U$$

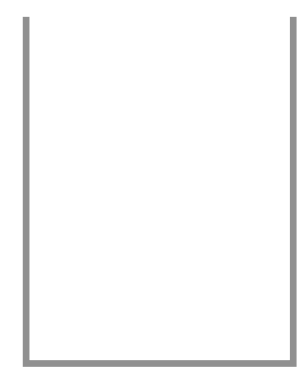
$$\hat{H} = \int dx \left(-\hat{\Psi}^\dagger \frac{\hbar^2}{2m}\nabla^2\hat{\Psi} + \hat{\Psi}^\dagger V_{ext}\hat{\Psi} + \frac{U}{2}\hat{\Psi}^\dagger\hat{\Psi}^\dagger\hat{\Psi}\hat{\Psi} \right)$$

Keeping quantum pressure term leads to “effective interaction” seen by inner observer:

$$\mathcal{U} = U - \frac{\hbar^2}{4mn_0}\left\{ \frac{(\nabla n_0)^2 - (\nabla^2 n_0)n_0}{n_0^2} - \frac{\nabla n_0}{n_0^2}\nabla + \nabla^2 \right\}$$

harmonic trap

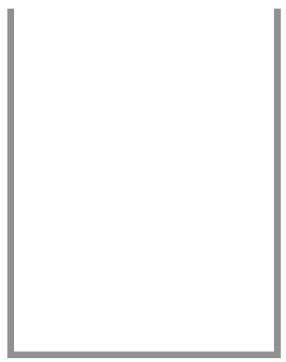
[position dependent sound speed]



condensate in box

[uniform number density]

Trans-Planckian beats signature



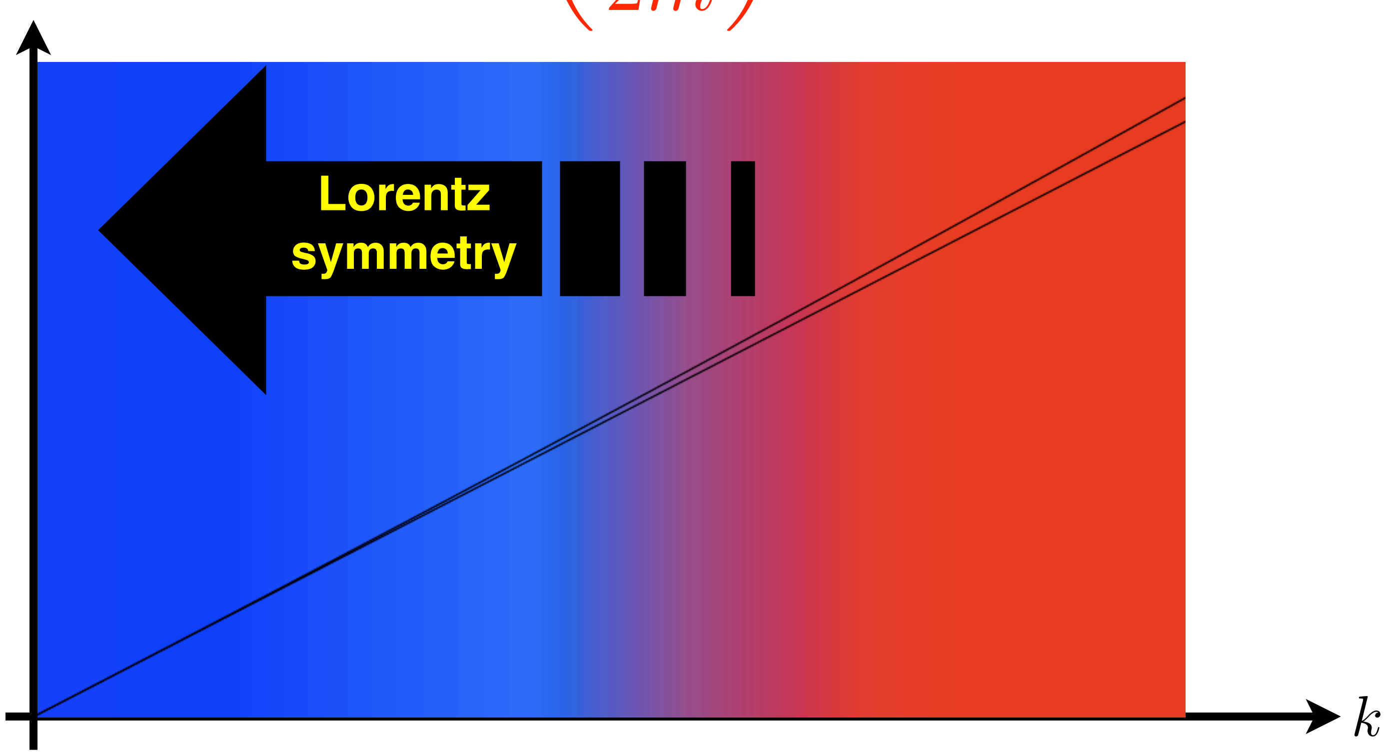
condensate in box

[uniform number density]

$$\mathcal{U} = U - \frac{\hbar^2}{4mn_0}\left\{\frac{(\nabla n_0)^2 - (\nabla^2 n_0)n_0}{n_0^2} - \frac{\nabla n_0}{n_0}\nabla + \nabla^2\right\} \longrightarrow \mathcal{U} = U - \frac{\hbar^2}{4mn_0}\nabla^2 .$$

$$c^2(U) = \frac{n_0 U}{m} \to c_k^2(\mathcal{U}) = c^2(U) + \left(\frac{\hbar}{2m}\right)^2 k^2$$

$$\omega_k^2 = c(t)\, k^2 + \left(\frac{\hbar}{2m}\right)^2 k^4$$

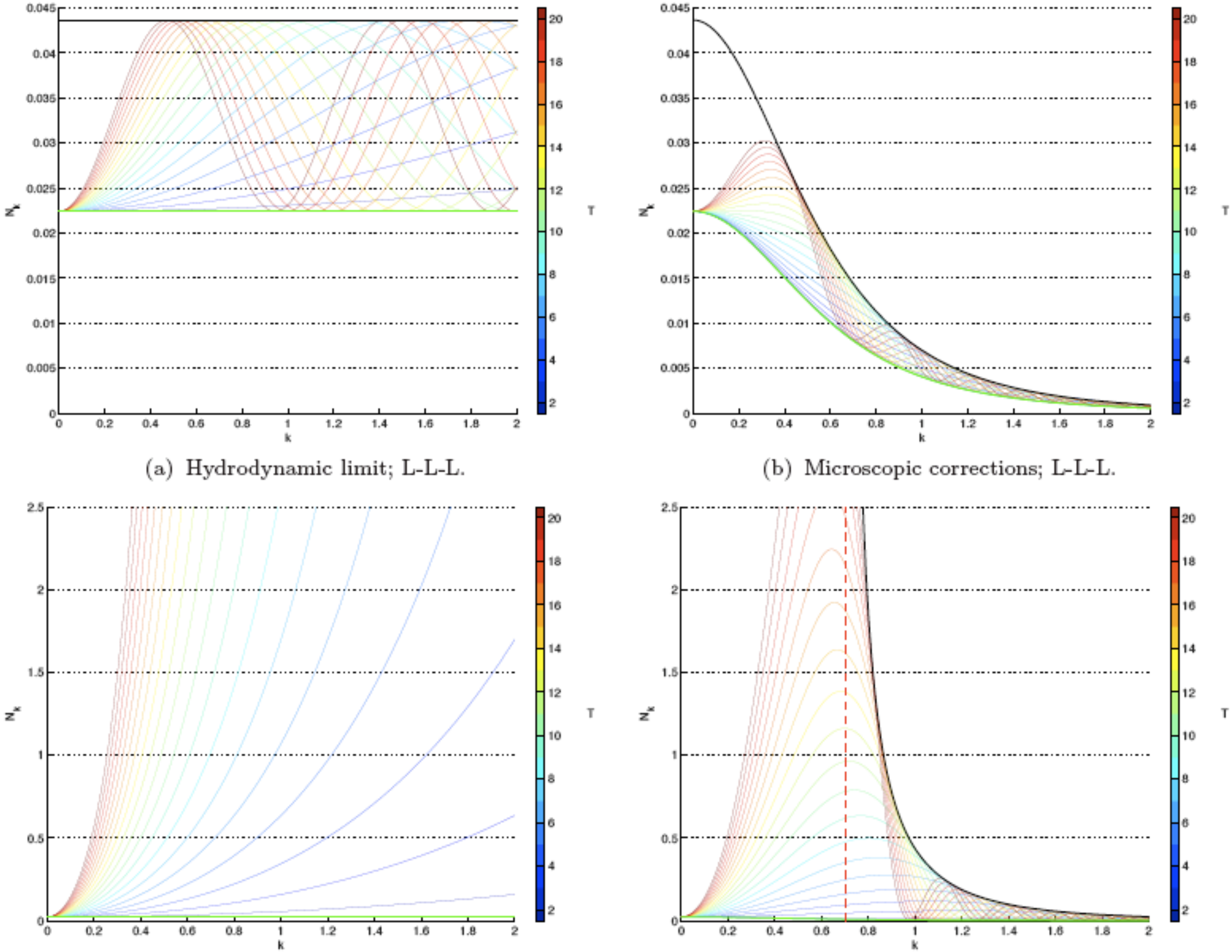


$$\omega_k \approx c(t)\, k \qquad \omega_k \approx \frac{\hbar}{2m} k^2$$

healing length:

$$\xi^2(t) = \left(\frac{\epsilon_{qp}}{c(t)}\right)^2 = \left(\frac{\hbar/2m}{c(t)}\right)^2$$

Trans-Planckian beats signature



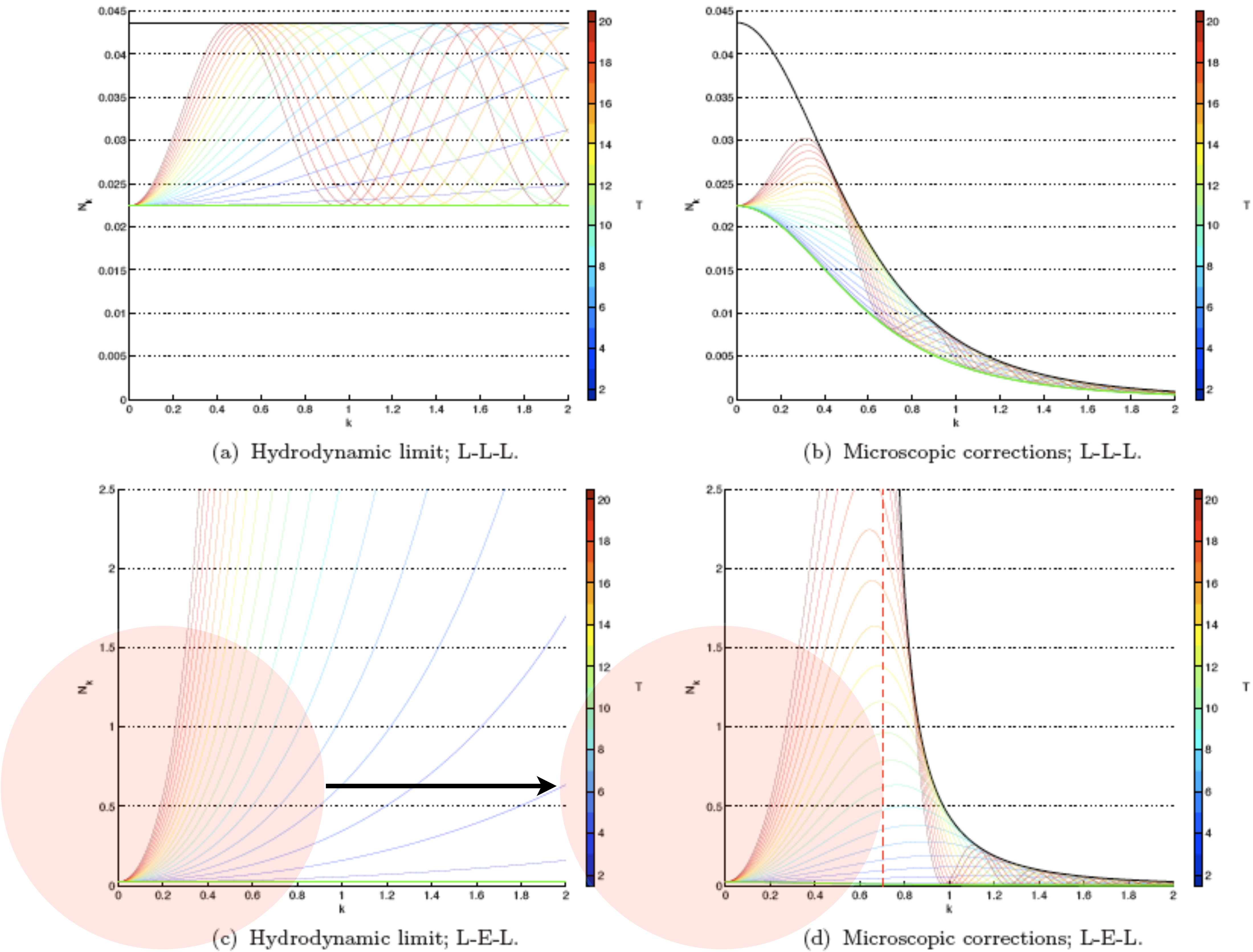
(a) Hydrodynamic limit; L-L-L.

(b) Microscopic corrections; L-L-L.

(c) Hydrodynamic limit; L-E-L.

(d) Microscopic corrections; L-E-L.

Trans-Planckian beats signature



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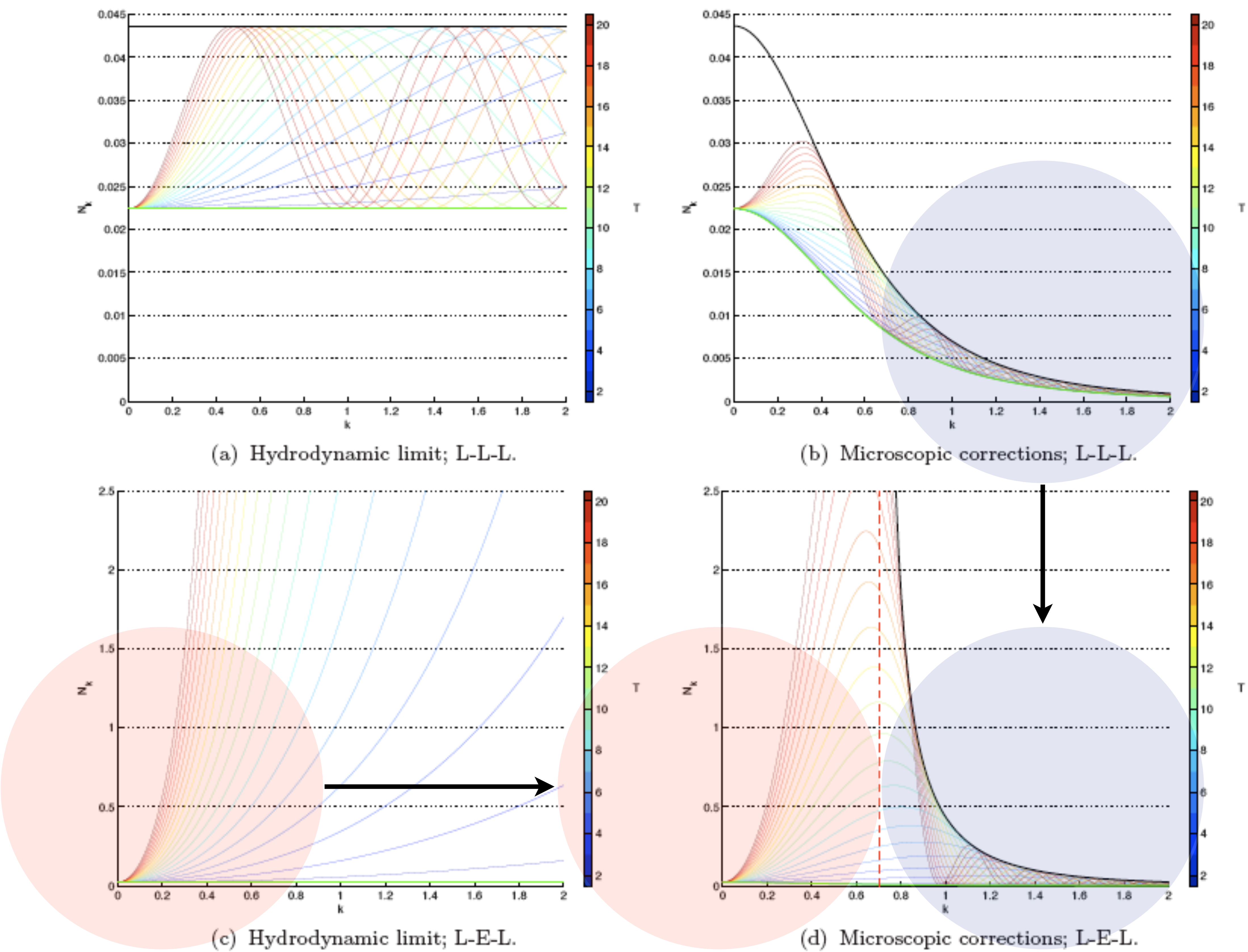

(a) Hydrodynamic limit; L-L-L.

(b) Microscopic corrections; L-L-L.

(c) Hydrodynamic limit; L-E-L.

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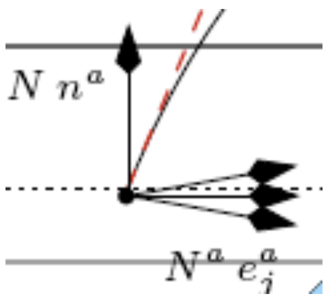
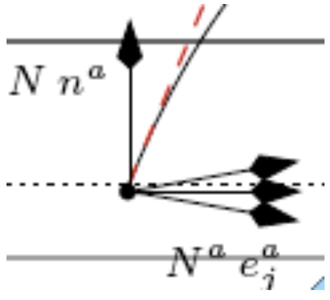
What about the *real* world?



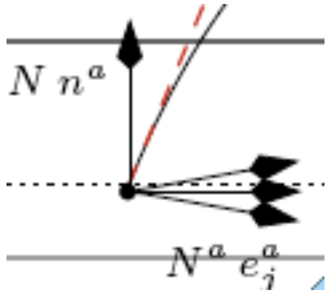
Signature of spacetime - where does it come from?

Signature of spacetime - what is it really?



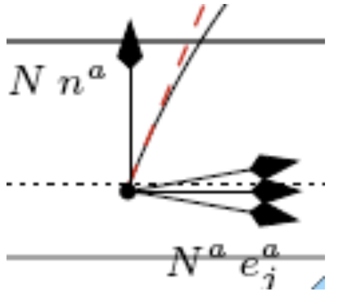
Signature of spacetime - what is it really?

- Signature of spacetime is a certain pattern of Eigenvalues of the metric tensor at each point of the manifold [Loretzian (-,+++) or Riemannian (+,+++)]



Signature of spacetime - what is it really?

- Signature of spacetime is a certain pattern of Eigenvalues of the metric tensor at each point of the manifold [Loretzian (-,+++) or Riemannian (+,+++)]
- Spacetime foliation into non-intersecting spacelike hypersurfacese (Lapse and Shift)



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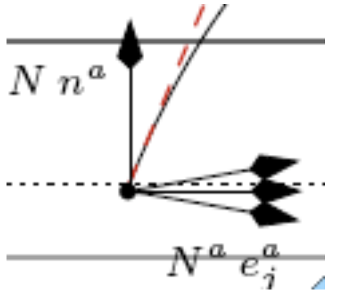
- Kinematics of signature change (Lapse is a non-dynamical variable), matching conditions

C. Teitelboim, "The Hamiltonian Structure Of Space-Time", General Relativity and Gravitation 1 (1981) 195–225.



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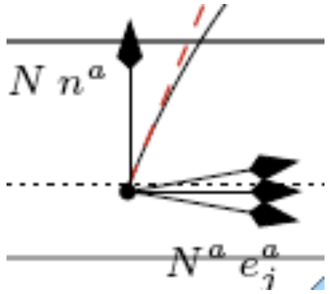
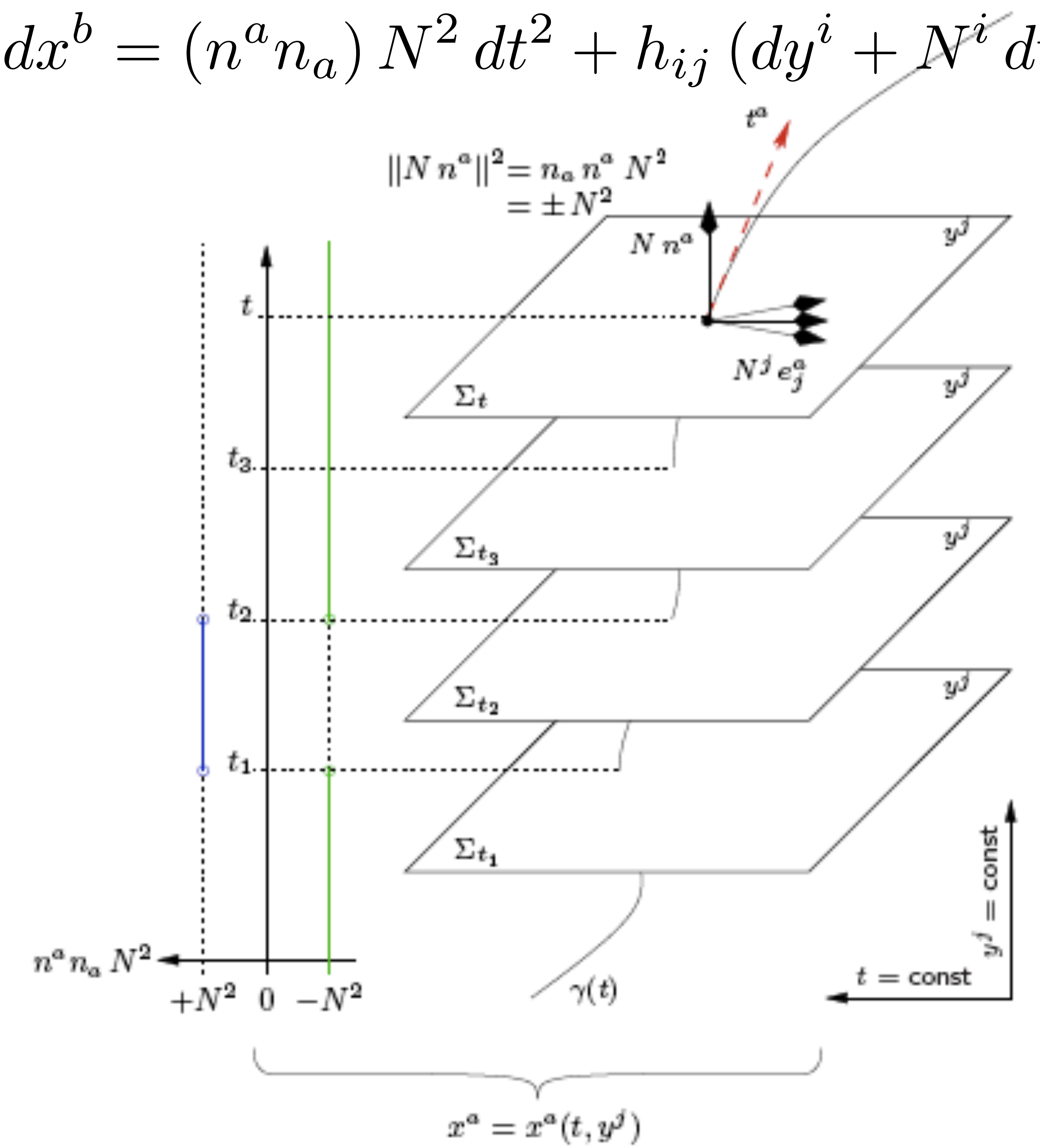
- There is no driving mechanism within GR that drives changes in the signature of the geometry...

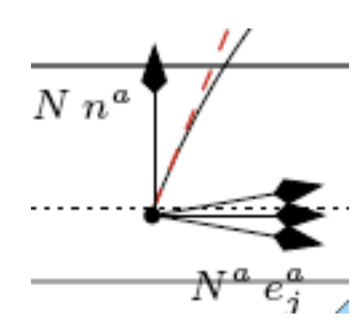


Signature of spacetime - what is it really?

- Spacetime foliation into non-intersecting spacelike hypersurfacese (Lapse and Shift)

$$ds^2 = g_{ab}\, dx^a dx^b = (n^a n_a)\, N^2\, dt^2 + h_{ij}\, (dy^i + N^i\, dt)\, (dy^j + N^j\, dt)$$





What about the non-dynamical side?

Quantum field theory on Riemannian manifolds

Quantum field theory on Riemannian manifolds

Spacetime kinematics:

FRW spacetimes, $k = 0$, shift vectors are zero, $N^j = 0$:

$$ds^2 = -B(t)\,dt^2 + A(t)\,d\vec{x}^2$$

We are interested in the case where $B = N^2$ alone changes sign:

$$- + + \cdots \rightarrow + + + \cdots \rightarrow - + + \ldots$$

Note that we can always use the coordinate freedom of general relativity make the signature change ``discontinuous''.

[1] G. Ellis, A. Sumeruk, D. Coule and C. Hellaby, "Change Of Signature In Classical Relativity", Class. Quant. Grav. **9** (1992) 1535.

[2] T. Dray, C. A. Manogue and R. W. Tucker, "Particle production from signature change", Gen. Rel. Grav. **23** (1991) 967.

T. Dray, C. A. Manogue and R. W. Tucker, "The Scalar field equation in the presence of signature change", Phys. Rev. D **48** (1993) 2587 [arXiv:gr-qc/9303002].

T. Dray, "General relativity and signature change", Contemporary Mathematics **359** (2004) 103–124.

Quantum field theory on Riemannian manifolds

QFT – canonical quantization scheme:

Lagrange density: $$\mathcal{L} = -\frac{1}{2}\sqrt{-g}\,\left(g^{ab}\partial_a\hat{\phi}(t,\mathbf{r})\,\partial_b\hat{\phi}(t,\mathbf{r}) + m^2\,\hat{\phi}(t,\mathbf{r})^2\right)$$

Conjugate momentum: $$\hat{\Pi}_\phi := \frac{\partial\mathcal{L}}{\partial\left(\partial_t\hat{\phi}\right)} = -\sqrt{-g}\,g^{tb}\partial_b\hat{\phi} \equiv -f^{tb}\partial_b\hat{\phi}$$

Equation of motion: $$\frac{1}{\sqrt{-g}}\partial_a\left(\sqrt{-g}\;g^{ab}\;\partial_b\hat{\phi}\right) = m^2\hat{\phi}$$

Continuity of field operators: $$\left[\hat{\phi}(t,\mathbf{r})\right] = 0 \quad \text{and} \quad \left[\hat{\Pi}_\phi(t,\mathbf{r})\right] = 0$$

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Quantum field theory on Riemannian manifolds

QFT – Bogliubov coefficients:

$$\hat{\phi}(t,\mathbf{r}) = \int \frac{d^{(d)}k}{(2\pi)^{d/2}} \frac{1}{\sqrt{2}} \left\{ u_k^*(t)\,\hat{a}_k + u_k(t)\,\hat{a}^\dagger_{-k} \right\}$$

Bogoliubov coefficients algebraically relate the mode functions beyond regions with different signatures:

$$\alpha = \left. \frac{\pi^*_{out} u_{in} - \pi_{in} u^*_{out}}{W_{out}} \right|_\Sigma \quad \text{and} \quad \beta = \left. \frac{\pi_{in} u_{out} - \pi_{out} u_{in}}{W_{out}} \right|_\Sigma$$

The mean number of *out*-particles in the *in*-vacuum is given by

$$\langle 0_{in} | \hat{a}^\dagger_{k,out} \hat{a}_{k,out} | 0_{in} \rangle = |\beta_k|^2\, \delta^d(0)$$

Quantum field theory on Riemannian manifolds

Effective equation of motion:

The only computational difficulty lies in finding the mode functions. It is convenient for our proposes to introduce auxiliary mode functions

$$u_k(t) := A(t)^{-d/4}\, v_k(t)$$

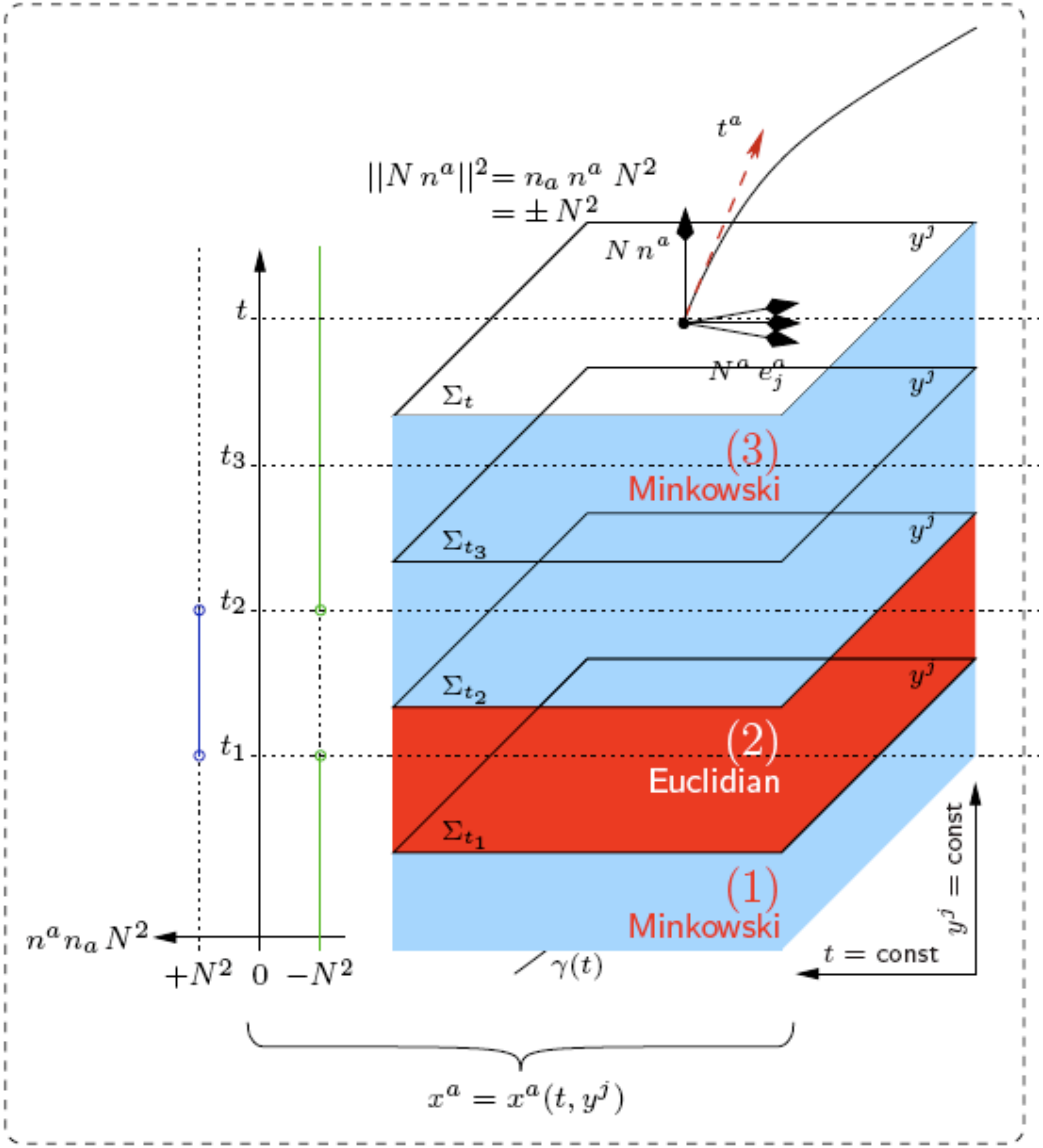
and its complex conjugate. (Notice, that here we are assuming $A(t) > 0$, and $B(t) = B$.) The wave equation for the auxiliary field in Fourier space reads

$$\ddot{v}_k(t) + \Omega_{\text{eff}}^2\, v_k(t) = 0,$$

where the effective frequency Ω_{eff} is given by

$$\Omega_{\text{eff}}^2 := \left(\frac{d}{4} - \frac{d^2}{4^2}\right) \frac{\dot{A}^2}{A^2} - \frac{d}{4}\,\frac{\ddot{A}}{A} + B\left(\frac{k^2}{A} + m^2\right).$$

Quantum field theory on Riemannian manifolds



Quantum field theory on Riemannian manifolds

Euclidean flat signature change events:

$$N_k = \sinh\left[\left(\frac{k^2}{A} + m^2\right)(t_2 - t_1)\right]^2$$

The quantum field modes in region $t < t_1$ and $t > t_2$ are given by

$$u_k = \frac{\exp(-i\omega_k t)}{A^{d/4}\sqrt{\omega_k}} \qquad \text{and} \qquad u_k^* = \frac{\exp(+i\omega_k t)}{A^{d/4}\sqrt{\omega_k}},$$

$$\pi_k = -iA^{d/4}\sqrt{\omega_k}\exp(-i\omega_k t) \qquad \text{and} \qquad \pi_k^* = +iA^{d/4}\sqrt{\omega_k}\exp(+i\omega_k t), \quad (1)$$

such that $W = \pi_k^* u_k - \pi_k u_k^* = 2i$, and $\omega_k^2 = \frac{k^2}{A} + m^2$.

Dispersion relation in intermediate regime:

$$\Omega^2_{\text{flat}} = -\left(\frac{k^2}{A} + m^2\right)$$

$$u_k = \frac{\cosh(|\Omega_{\text{flat}}|t) + i\,\sinh(|\Omega_{\text{flat}}|t)}{A^{d/4}\sqrt{|\Omega_{\text{flat}}|}} \qquad \text{and} \qquad \text{c.c.};$$

$$\pi_k = -i\,A^{d/4}\,\sqrt{|\Omega_{\text{flat}}|}\,\left(\cosh(|\Omega_{\text{flat}}|t) - i\,\sinh(|\Omega_{\text{flat}}|t)\right) \qquad \text{and} \qquad \text{c.c.}\,.$$

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Physical grasp on quantum field on Riemannian manifolds – rectangular barrier:

The transmission coefficient for quantum tunneling through a rectangular barrier is given by

$$T = \frac{4E(V-E)}{4E(V-E) + V^2 \sinh^2[\sqrt{2m(V-E)}\, L]},$$

where V is the height of the barrier, L is its width, and E is the incident energy. If we now take the special case $E = \frac{1}{2}V$ we have

$$T = \frac{1}{1+\sinh^2[\sqrt{2m(V-E)}\, L]} = \frac{1}{\cosh^2[\sqrt{2m(V-E)}\, L]},$$

from which, using the standard equivalences

$$|\alpha|^2 \leftrightarrow 1/T \qquad \text{and} \qquad |\beta|^2 \leftrightarrow (1-T)/T = R/T,$$

we see

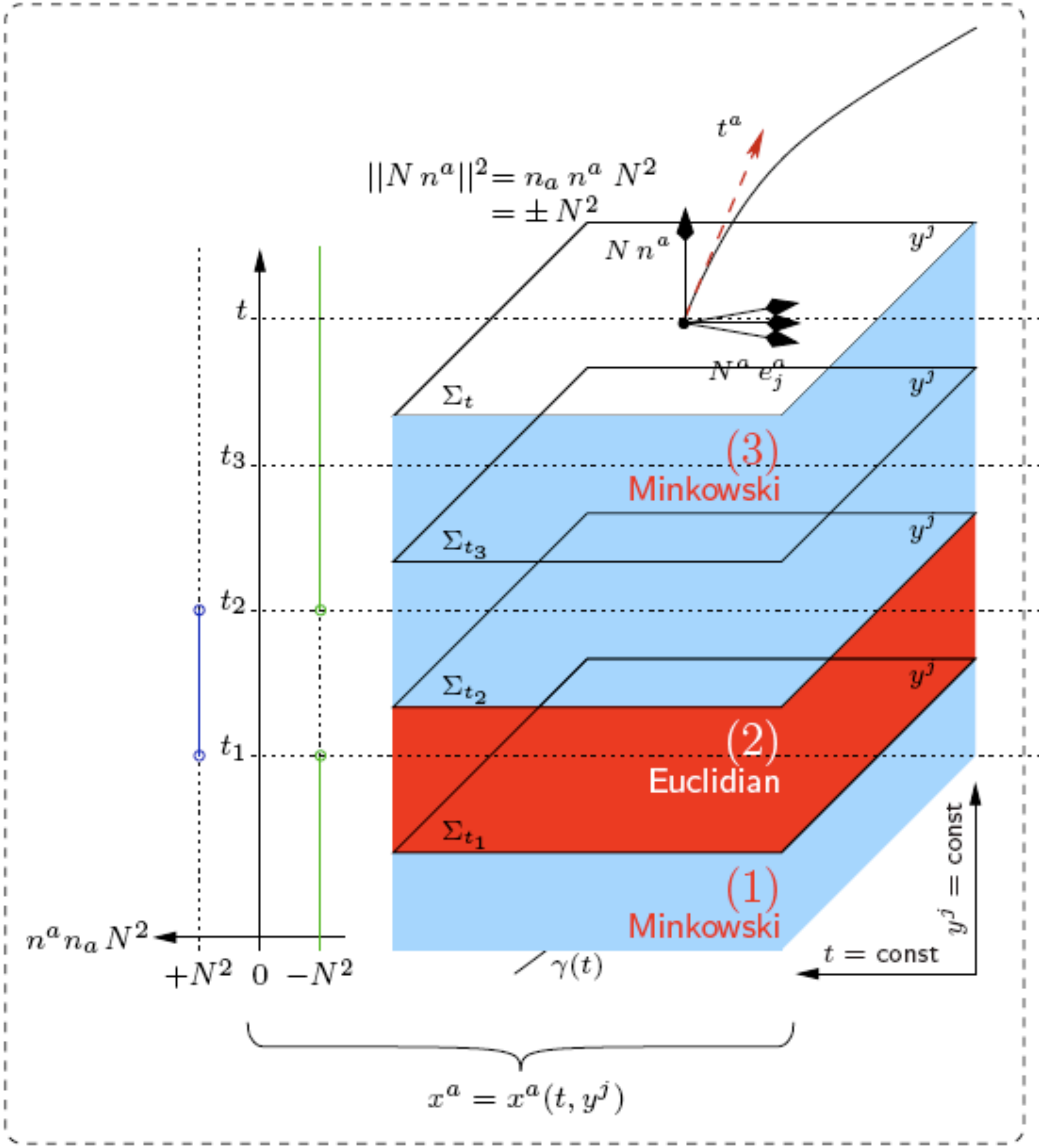
$$|\alpha|^2 \leftrightarrow \cosh^2[\sqrt{2m(V-E)}\, L] = \cosh^2[\kappa\, L] \leftrightarrow \cosh^2\{\, |\bar{\omega}_{\text{effective}}|\ \Delta\tau\},$$

that is

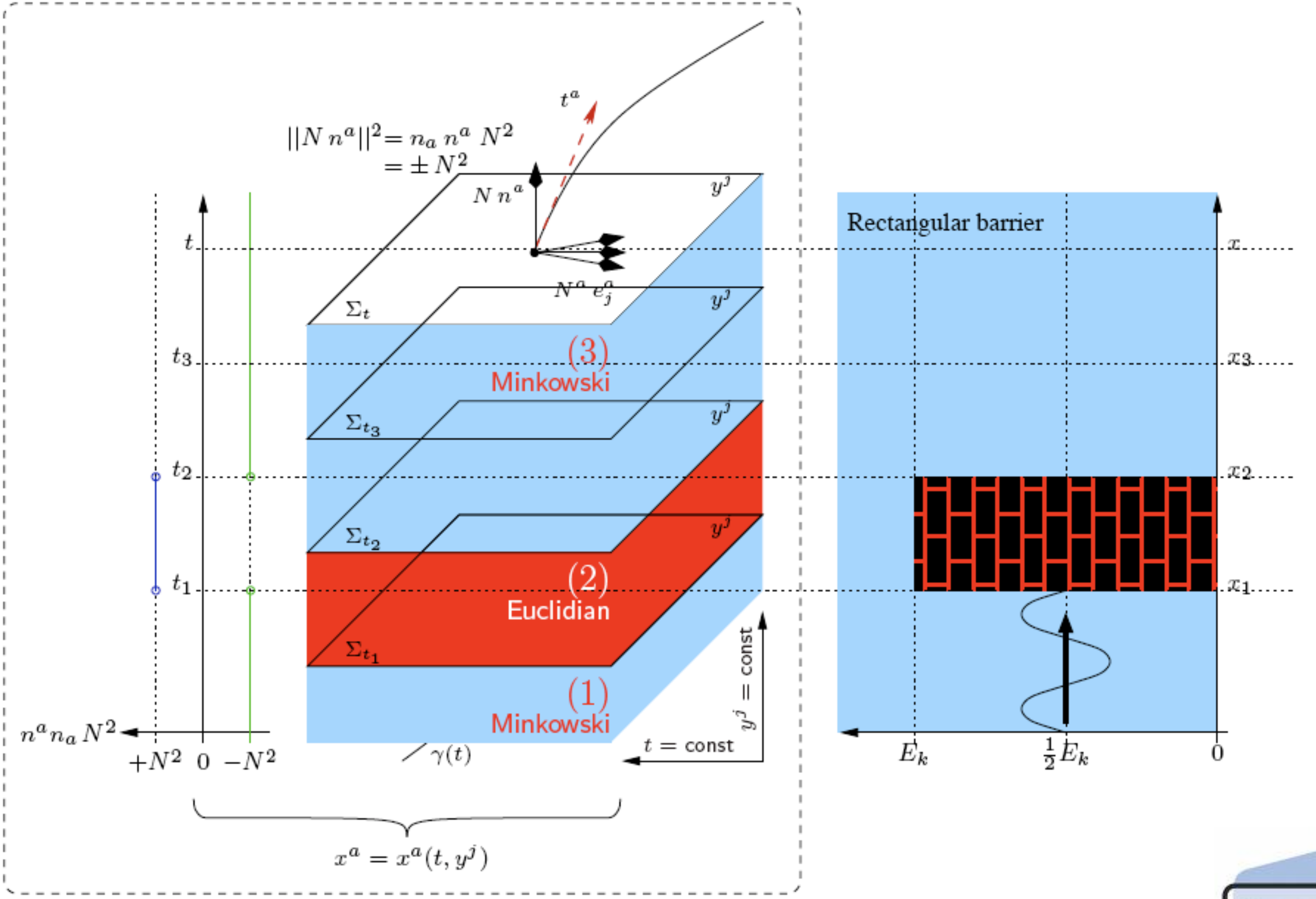
$$|\alpha| \leftrightarrow \cosh\{\, |\bar{\omega}_{\text{effective}}|\ \Delta\tau\},$$

completely in agreement with our exact calculation.

Quantum field theory on Riemannian manifolds



Quantum field theory on Riemannian manifolds



Physical grasp on quantum field on Riemannian manifolds – super-Hubble horizon modes in cosmology:

Mechanism responsible for enormous particle production works analogous to cosmological particle production during inflation:

$$\ddot{v}_k(t) + \Omega_{\text{eff}}^2 \, v_k(t) = 0,$$

$$\Omega_{\text{flat}}^2 = -\left(\frac{k^2}{A} + m^2\right)$$

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$$\ddot{\hat{\chi}}_k(t) + \left(\frac{k^2}{e^{2Ht}} + m^2 - \frac{d^2 H^2}{4}\right)\hat{\chi}_k(t) = 0$$

$$m < d\,\frac{H}{2}$$

$$k < k_{\text{HubbleHorizon}}$$

$$\ddot{v}_k(t) + \Omega^2_{\text{eff}}\, v_k(t) = 0,$$

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Quantum field theory on Riemannian manifolds

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frozen modes

$$\ddot{v}_k(t) + \Omega^2_{\mathrm{eff}}\, v_k(t) = 0,$$

$$\Omega^2_{\mathrm{flat}} = -\left(\frac{k^2}{A} + m^2\right)$$

all modes are *frozen*

Significant particle production on ALL scales!???

QFT

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There is always hope...

Trans-Planckian beats signature?

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Let’s do quantum gravity phenomenology, in the sense of an ultra-high energy breakdown of Lorentz symmetry

T. Jacobson, “Black hole evaporation and ultrashort distances”, Phys. Rev. D 44 (1991) 1731

$$\Delta_{d+1}\phi - F(-\Delta_d)\phi = m^2\phi$$

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$$\Delta_d\,\phi = \frac{1}{\sqrt{g_d}}\partial_i\left(\sqrt{g_d}\;g^{ij}\;\partial_j\phi\right)$$

$$\bar{\omega}^2_{\text{effective}} = \epsilon A^d\left[m^2 + F(k^2/A) + k^2/A\right]$$

$$\beta \approx i\sinh\left\{\int_E \sqrt{m^2 + k^2/A + F(k^2/A)}\;A^{d/2}\;d\bar{t}\right\}$$

Trans-Planckian beats signature

Particle production in *real* world with naive LIV terms:

$$\beta \approx i \sinh \left\{ \int_E \sqrt{m^2 + k^2/A + F(k^2/A)}\, A^{d/2}\, d\bar{t} \right\}$$

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Particle production in analogue *world* - a BEC - with quantum pressure correction to the mean-field:

$$\beta \approx i \sin\left\{\int_E \sqrt{B\, m^2 + \varepsilon_{\rm qp}^2\, k^4 + B\, k^2/A}\; A^{d/2}\; d\bar{t}\right\}$$

So what…

What, **if** anything at all, did we learn from this!?

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* classically signature change events are closely related to quantum tunneling "half-way up" the barrier

* quantum mechanically signature change events are closely super-Hubble horizon modes during inflation

* Signature change events in the *real* universe show serious problems: driving the production of an infinite number of particle, with infinite energy, which are not removed by dimension, rest mass, or even resonable sub-class of LIV

What, if anything at all, did we learn from this?

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If there is a way to drive sig. change events within the realm QG, there should be a mechanism to regularize the infinities..! (Analogue to the situation in the BEC)

The end...

books on the **Emergent Spacetime** / **Analogue Models** for Gravity:

Artificial black holes (Novello, Visser, Volovik)

The universe in a helium droplet (Volovik)

Quantum Analogues: From phase transitions to black holes and cosmology (Unruh, Schuetzhold)

I'd like to thank David and Matt!