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Small, dark, and heavy... but is it a black hole ?

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Abstract:

Astronomers have certainly seen things that are small, dark, and heavy...

But are these small, dark, heavy objects really black holes in the sense of general relativity **?**

(The consensus opinion is simply "yes", and there is very little "wriggle room".)

I'll discuss some of the alternatives...

Do black holes "exist"?

Observational astronomy:

Small, dark, and heavy...

Accretion disks probe down to the ISCO:

$2m/r \sim$ **1/3 !**

ADAFs probe down to $2m/r \sim$ **1 ?**

Everything so far compatible with Schwarzschild/ Kerr.

Do black holes "exist"?

Theory:

(Eternal) black holes certainly exist mathematically, as stationary vacuum solutions in general relativity...

Classical black holes (future event horizons) certainly exist mathematically as the end result of classical collapse based on certain physically plausible equations of state.

BUT...

Do black holes "exist"?

Can one avoid black hole formation with a suitably weird equation of state **?**

Can one avoid black hole formation with semi-classical quantum effects **?**

Can one avoid black hole formation with "quantum gravity" **?**

The possibilities are rather tightly constrained.

Do black holes "exist"?

There is of course the utter gibbering crackpot fringe...

(Names suppressed to protect the guilty.)

"Physically reasonable" alternatives to black hole formation are counted on the fingers of one (severely mutilated) hand...

(For selected values of "physically reasonable".)

Do alternatives “exist”?

Quark stars, Q-balls, boson-stars?

Gravastars: Mazur--Mottola variants.

Gravastars: Laughlin-et-al variants.

Fuzz-balls: Mathur-et-al variant.

Fuzz-balls: Amati variant.

Vachaspati --- Krauss... Hacijek...

Boulware... Marek Abramowicz...

Quark stars, Q-balls, boson-stars?

(Change EOS: Star/white dwarf/ neutron star/ etc...)

Questionable justification for EOS...

Still have Buchdahl-Bondi bound:
$2m/r >= 8/9$ for any
isotropic pressure profile.

So you cannot get "close" to $2m/r \sim \mathbf{1}$,
unless you have anisotropic stresses.

Gravastars:

Core: de Sitter like....

Exterior: Schwarschild like...

Where the horizon would have formed: $2m/r \sim \mathbf{1}$

1) don't ask...

2) anisotropies guaranteed...

3) breakdown of spacetime manifold **?** [Laughlin]

4) one-loop action **?** [Mazur--Mottola]

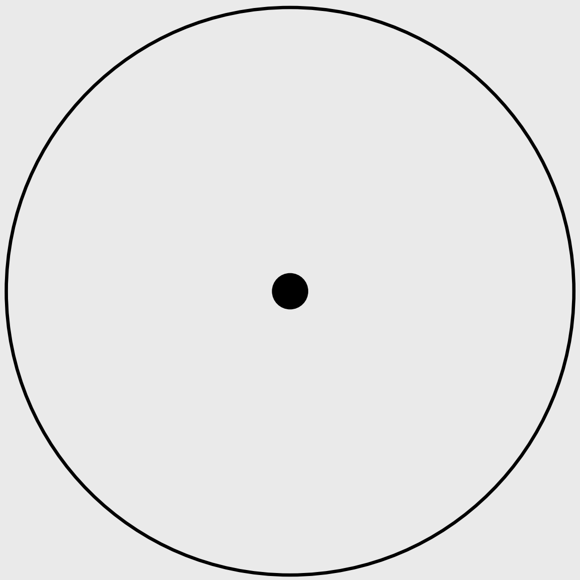
Fuzz balls:

Explicit calculations appear to be limited to extremal/ near-extremal regime...

Black hole "interior" = "string muck"?

Not *a* spacetime, a *superposition* of "spacetimes"?

(And none of the individual "spacetimes" in the superposition has a horizon?)



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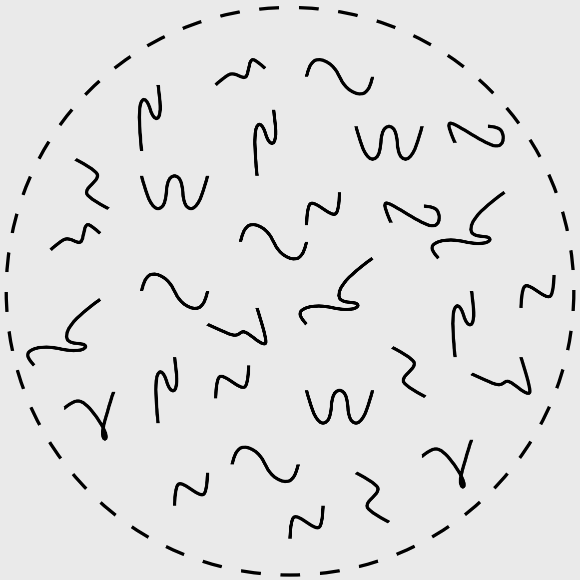

Fuzz ball

Our proposal:

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Fate of gravitational collapse in semiclassical gravity.

Carlos Barcelo, Stefano Liberati,
Sebastiano Sonego, Matt Visser.

e-Print: arXiv:0712.1130 [gr-qc]
Physical Review D77 (2008) 044032

Our proposal:

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Related to questions of the (quantum) vacuum...

*** Boulware vacuum? (singular at any Killing horizon)

Renormalized stress-energy diverges at 2m/r ~ **1**

*** Unruh vacuum? (designed to be well behaved at any future horizon)

Renormalized stress energy finite at future horizon.

Questions:

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Unruh vacuum is super-teleological:

(This is the "standard picture" perpetrated in Bill's name, though Bill himself might choose to disagree...)

Somehow the star has to "know", before its collapse, that it *will* collapse sometime in the future... and somehow conspire to be in the appropriate Unruh quantum vacuum state...

Somehow stars that never collapse also "know" this, and somehow conspire to be in the appropriate Boulware quantum vacuum state...

Issues:

(Names again suppressed to protect the guilty...)

Technically, it's not just that the Boulware and Unruh vacuum states themselves are orthogonal, but more critically the entire Fock spaces built on the Boulware and Unruh vacuum states are orthogonal, and the star somehow has to choose, ahead of time, which of these Fock spaces it is living in...

(Delayed choice, but delayed until the heat death **?**)

Issues:

Fulling--Sweeny--Wald (no-singularity) theorem:

CMP 63 (1978) 257-264.

Loosely: "Everything in curved-spacetime QFT is hunky-dory at the event horizon, and all the way down to either the singularity or Cauchy horizon..."

Based on showing that the Hadamard form of the QFT two-point function is not affected by the presence of an event horizon...

(So if you choose a Hadamard state, everything is fine...)

Issues:

Unfortunately, Fulling--Sweeny--Wald also "begs the question"...

FSW shows that *if* an event horizon forms, then the QFT can be forced to be well behaved there...

This is *not* the same as showing that an event horizon will naturally form in semiclassical collapse...

Finite <=/=> "small"...

Compact horizonless objects, and/or naked singularities, are also compatible with the FSW theorem.

Issues:

If you believe that Hawking evaporation is unitary:

(as seen from our own asymptotically flat region...)

"The way the information gets out seems
to be that a true event horizon never forms,
just an apparent horizon."

(Stephen Hawking in the abstract to his GR17 talk.)

The event/ absolute/ apparent/ trapping/ horizon distinction may be critically important...

Uncollapsed state:

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Star before collapse:

$$G_{\mu\nu} = 8\pi \left(T^{c}_{\mu\nu} + \langle C | \widehat{T}_{\mu\nu} | C \rangle \right) \ ,$$

Vacuum polarization effect negligible in an ordinary uncollapsed star...

(This, after all, is why we can get away with just solving the classical Einstein equations most of the time...)

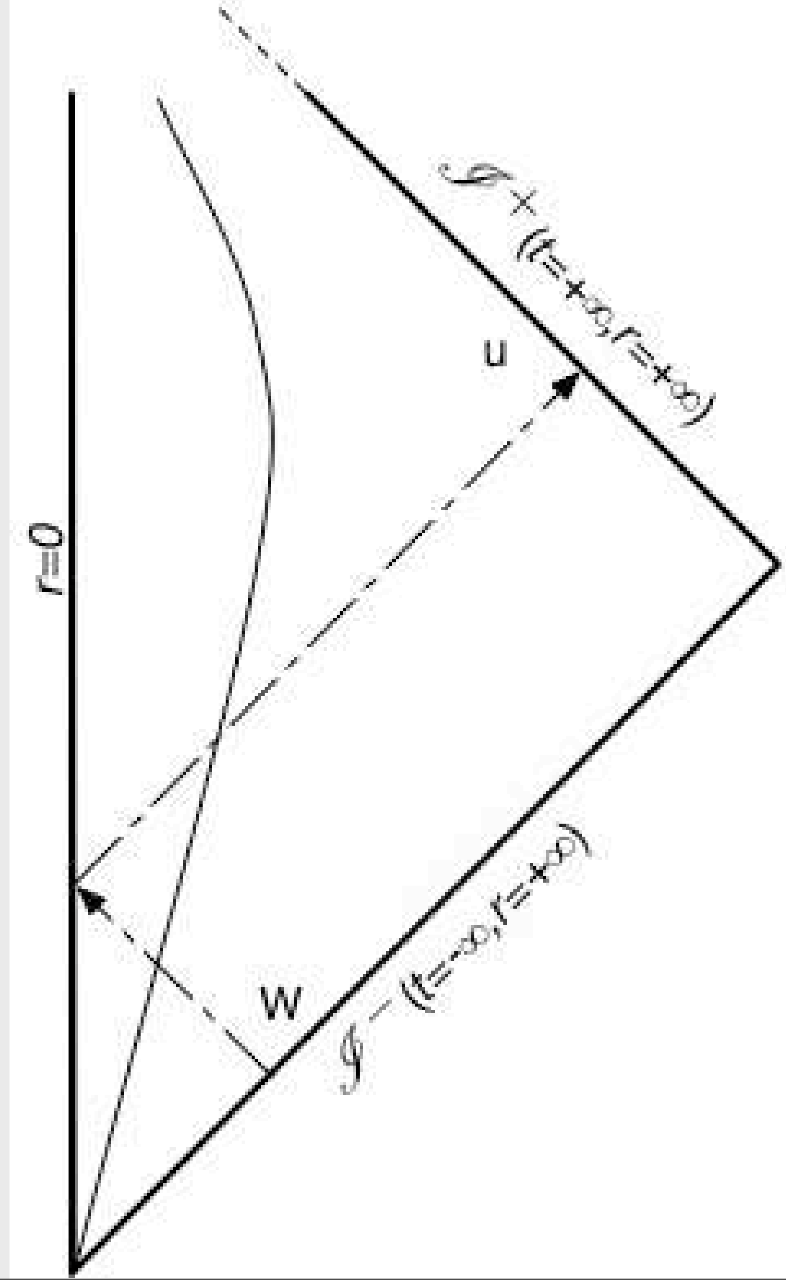
Does this remain true during collapse?

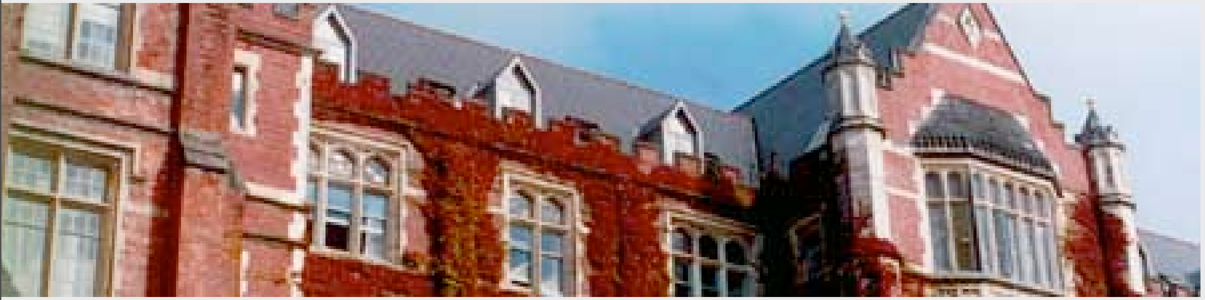
Collapse picture:

Standard collapse picture:

(Modes "bounce" off the centre...)

(**1+1**) dimensions to keep calculation tractable...





Collapse picture:

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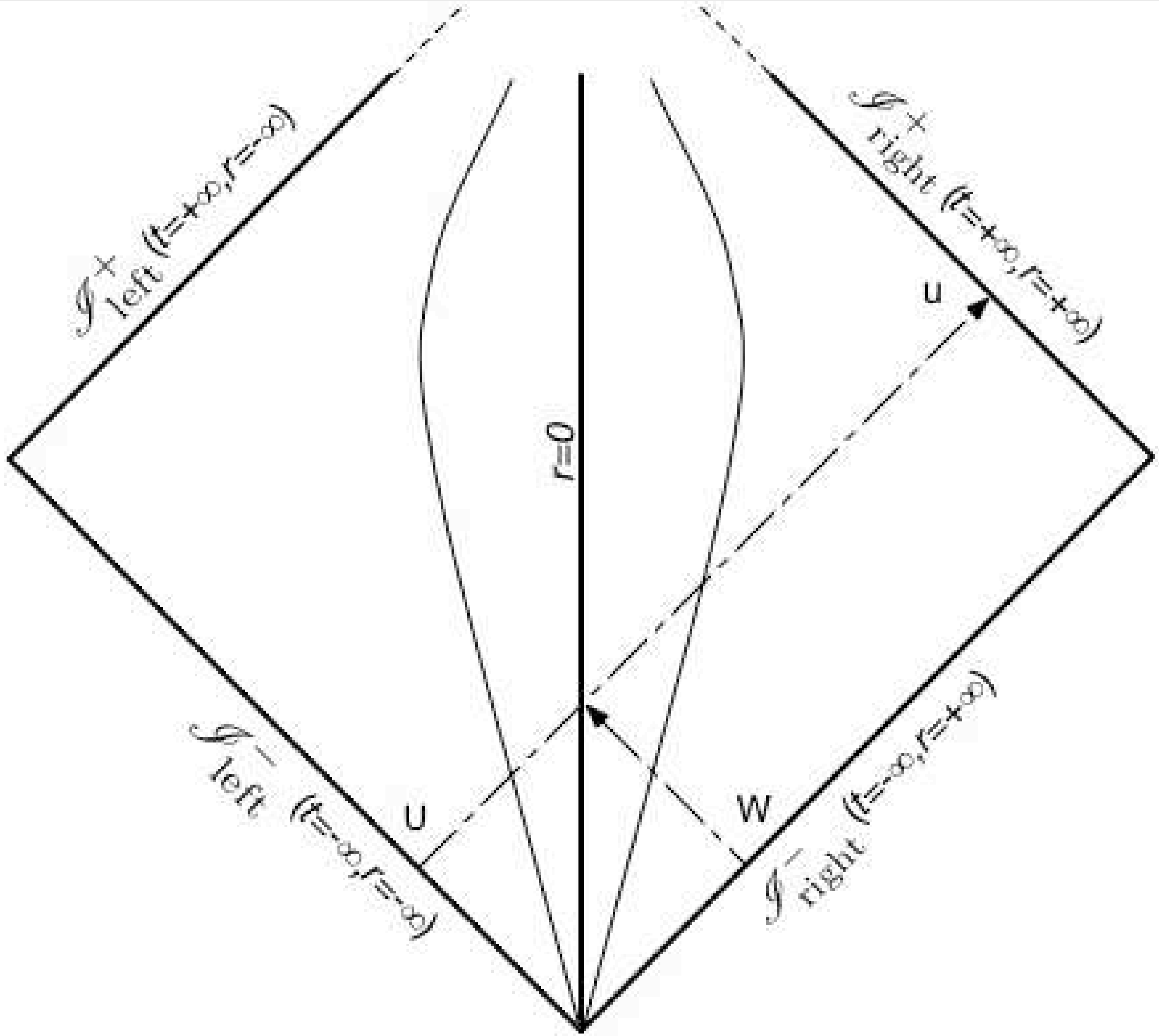
Our preferred "symmetric" version...

Modes propagate straight through the centre...

Affine null coordinates:

$$U = p(u)$$



$$U = p(u)$$

Choose modes:

near $\mathscr{I}^-$

$$\varphi_\Omega(r,t) \approx \frac{1}{(2\pi)^{3/2}(2\Omega)^{1/2}\,|r|}\,\mathrm{e}^{-i\Omega U}\;,$$

near $\mathscr{I}^+$

$$\varphi_\Omega(r,t) \approx \frac{1}{(2\pi)^{3/2}(2\Omega)^{1/2}\,r}\,\mathrm{e}^{-i\Omega p(u)}\;.$$

$$U = p(u)$$

but we could also write: near $\mathscr{I}^+$

$$\psi_\omega(r,t) \approx \frac{1}{(2\pi)^{3/2}(2\omega)^{1/2}\, r}\, \mathrm{e}^{-i\omega u} \,,$$

this lets you define:

$$\omega(u,\Omega) = \dot{p}(u)\ \Omega,$$

adiabatic condition:

$$|\dot{\omega}(u,\Omega)|/\omega^2 \ll 1$$

Collapse picture:

"Modes" are excited if adiabatic condition fails.

This occurs at:

$$\Omega_0(u) \sim |\ddot{p}(u)|/\dot{p}(u)^2 \; .$$

One can then think of $\Omega_0(u)$ as a frequency marking, at each instant of retarded time u, the separation between the modes that have been excited ($\Omega \ll \Omega_0$) and those that are still unexcited ($\Omega \gg \Omega_0$).

Collapse estimate:

Naively, you can think of an infinite "reservoir" of Boulware-like modes above:

$$\Omega_0(u) \sim |\ddot{p}(u)|/\dot{p}(u)^2 \ .$$

Contributing to the RSET:

$$\langle 0_B|\widehat{T}_{\hat{\mu}\hat{\nu}}(r)|0_B\rangle_{\rm ren} \propto -\frac{1}{M^2}\,\frac{1}{1-2M/r}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

This is, however, far too naive a picture, instead, let us calculate...

Collapse calculation:

Metric and coordinates:

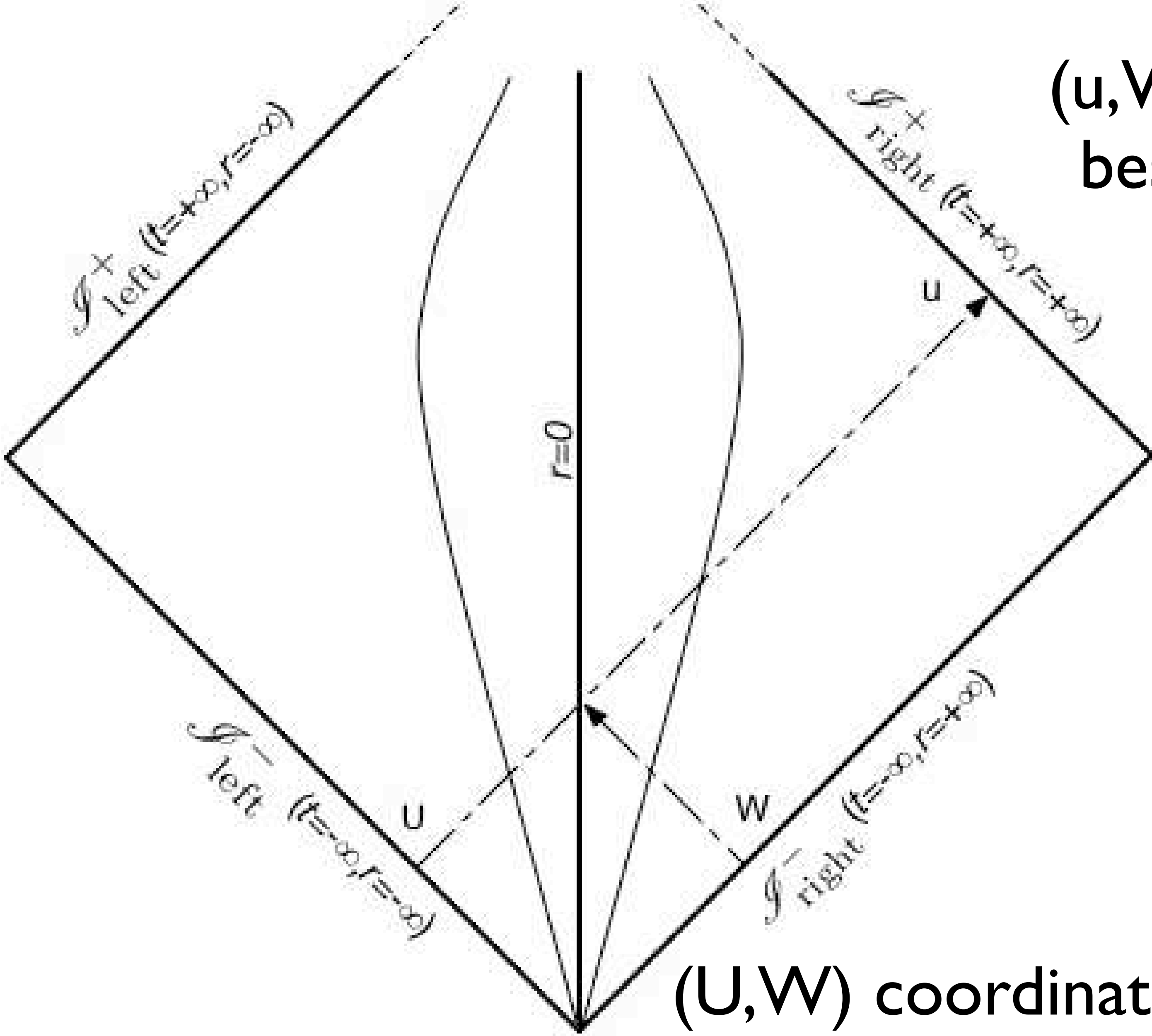
$$g = -C(U,W)\,\mathrm{d}U\,\mathrm{d}W \ .$$ (scri-, scri-)

$$g = -\bar{C}(u,W)\,\mathrm{d}u\,\mathrm{d}W \ .$$ (scri+, scri-)

$$C(U,W) = \bar{C}(u,W)/\dot{p}(u) \ ,$$

$$\partial_U = \dot{p}^{-1}\,\partial_u \ .$$

Collapse calculation:



(u,W) coordinates best near here...

(U,W) coordinates best near here...

Collapse calculation:

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RSET:

$$T_{UU} \propto C^{1/2}\, \partial_U^2\, C^{-1/2} \; ,$$

$$T_{WW} \propto C^{1/2}\, \partial_W^2\, C^{-1/2} \; ,$$

$$T_{UW} \propto R \; .$$

Quantum state "initially Boulware-like".

Specific coefficients not particularly important...

T_{WW} and T_{UW} are automatically OK.

T_{UU} needs a calculation...

Collapse calculation:

Key point:

$$C^{1/2}\,\partial_U^2\,C^{-1/2} = \frac{1}{\dot{p}^2}\left[\bar{C}^{1/2}\,\partial_u^2\,\bar{C}^{-1/2} - \dot{p}^{1/2}\,\partial_u^2\,\dot{p}^{-1/2}\right].$$

"static" contribution

"dynamic" contribution

Boulware-like...

excited modes...

(static spacetime outside collapsing star)

[can arrange cancellations...]

Collapse calculation:

The key point here is that we have two terms, one $(\bar{C}^{1/2}\,\partial_u^2\,\bar{C}^{-1/2})$ arising purely from the static spacetime outside the collapsing star, and the other $(\dot{p}^{1/2}\,\partial_u^2\,\dot{p}^{-1/2})$ arising purely from the dynamics of the collapse. If, and only if, the horizon is assumed to form at finite time will the leading contributions of these two terms cancel against each other — this is the standard scenario.

Collapse calculation:

Indeed the first term is exactly what one would compute from using standard Boulware vacuum for a static star. As the surface of the star recedes, more and more of the static spacetime is “uncovered”, and one begins to see regions of the spacetime where the Boulware contribution to the RSET is more and more negative, in fact diverging as the surface of the star crosses the horizon.

Painleve--Gullstrand:

$$g = -c^2(x,t)\,\mathrm{d}t^2 + [\,\mathrm{d}x - v(x,t)\,\mathrm{d}t\,]^2\;.$$

Technical computation:

$$\begin{aligned}
T_{tt} &= U_t^2\,T_{UU} + 2\,U_t\,W_t\,T_{UW} + W_t^2\,T_{WW} \\
&= (c+v)^2\,U_x^2\,T_{UU} - 2\,(c^2 - v^2)\,U_x\,W_x\,T_{UW} + (c-v)^2\,W_x^2\,T_{WW} \\
&= \dot{p}^2\,T_{UU} - 2\,\dot{p}\,T_{UW} + T_{WW}\;;
\end{aligned}$$

$$\begin{aligned}
T_{tx} &= U_t\,U_x\,T_{UU} + (U_t\,W_x + U_x\,W_t)\;T_{UW} + W_t\;W_x\;T_{WW} \\
&= -(c+v)\,U_x^2\,T_{UU} - 2\,v\,U_x\,W_x\,T_{UW} + (c-v)\,W_x^2\,T_{WW} \\
&= -\frac{\dot{p}^2}{c+v}\,T_{UU} + \frac{2\,\dot{p}\,v}{c^2-v^2}\,T_{UW} + \frac{1}{c-v}\,T_{WW}\;;
\end{aligned}$$

$$\begin{aligned}
T_{xx} &= U_x^2\,T_{UU} + 2\,U_x\,W_x\,T_{UW} + W_x^2\,T_{WW} \\
&= \frac{\dot{p}^2}{(c+v)^2}\,T_{UU} - 2\,\frac{\dot{p}}{c^2-v^2}\,T_{UW} + \frac{1}{(c-v)^2}\,T_{WW}\;.
\end{aligned}$$

Collapse calculation:

Calculation assuming normal horizon formation

Static contribution:

$$v(x) \approx -1 + \kappa\, x + \kappa_2\, x^2 + \cdots$$

$$\bar{C}^{1/2}\, \partial_u^2\, \bar{C}^{-1/2} = \frac{\kappa^2}{4} + \mathcal{O}(x^2).$$

Collapse:

$$p(u) \approx U_{\rm H} - A_1\, {\rm e}^{-\kappa u}$$

Note!

$$p(u) = U_{\rm H} - A_1\, {\rm e}^{-\kappa u} + \frac{A_2}{2}\, {\rm e}^{-2\kappa u} + \frac{A_3}{3!}\, {\rm e}^{-3\kappa u} + \cdots$$

$$p(u) = U_{\rm H} - F({\rm e}^{-\kappa u})$$

Dynamic contribution:

$$\dot{p}^{1/2}\,\partial_u^2\,\dot{p}^{-1/2} = -\frac{1}{2}\frac{\dddot{p}}{\dot{p}} + \frac{3}{4}\left(\frac{\ddot{p}}{\dot{p}}\right)^2$$

$$= \frac{\kappa^2}{4} + \left[-\frac{1}{2}\frac{F'''}{F'} + \frac{3}{4}\left(\frac{F''}{F'}\right)^2\right]\kappa^2\,\mathrm{e}^{-2\kappa u}$$

$$= \frac{\kappa^2}{4} + \left[-\frac{1}{2}\frac{A_3}{A_1} + \frac{3}{4}\left(\frac{A_2}{A_1}\right)^2\right]\kappa^2\,\mathrm{e}^{-2\kappa u}$$

$$+\mathcal{O}\left(\mathrm{e}^{-3\kappa u}\right) \; .$$

?

Note: Leading term cancels against static contribution...

Dynamic contribution:

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Calculation assuming asymptotic horizon formation

$$r(t) = 2M + B\mathrm{e}^{-\kappa_\mathrm{D} t}$$ (PG coords)

$$p(u) = U_\mathrm{H} - A_1 \mathrm{e}^{-\kappa_\mathrm{eff}\, u}$$

$$\kappa_\mathrm{eff} = \frac{\kappa\ \kappa_\mathrm{D}}{\kappa + \kappa_\mathrm{D}} \qquad \kappa_\mathrm{eff} < \kappa.$$

(Still get Hawking-like radiation...; no true horizon...)

Dynamic contribution:

Now only have partial cancellation outside the star:

$$\mathrm{RSET}(x \approx 0) \approx \frac{1}{\kappa^2 x^2}\left(\kappa_{\mathrm{eff}}^2 - \kappa^2\right)$$

$$= -\frac{\kappa\,(2\,\kappa_{\mathrm{D}} + \kappa)}{\left(\kappa_{\mathrm{D}} + \kappa\right)^2\; x^2},$$

Does not violate FSW (finite <=/=> small)

RSET can become large (albeit finite in compliance with FSW) as one approaches 2m/r ~ **1**

Collapse calculation:

- In the standard collapse scenario the regularity of the RSET at horizon formation is due to a subtle cancelation between the dynamical and the static contributions.

- Contributions that can be neglected at late times can indeed be very large at the onset of horizon formation. The actual value of these contributions depends on the rapidity with which the configuration approaches its trapping horizon.

Collapse scenario:

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QUASI-BLACK HOLE SCENARIO

In the standard collapse you can argue that the RSET at horizon-crossing felt by infalling matter is negligible if you have:

1) a Hadamard state (which we have by assertion --- FSW)
2) matter is basically free falling
3) the equivalence principle holds

The first point tells you that the quantum vacuum has the same UV form as in Minkowski spacetime, the second point tells you that matter is approximately in a local inertial frame, the third point tells you that the local RSET the matter then "feels" must be approximately the value it has in Minkowski spacetime,
i.e. approximately zero (after renormalization).

Collapse scenario:

Our result is saying exactly that large deviations from this standard conclusion can arise if matter is not freely falling, but actually accelerated (as it must be to sustain itself against the gravitational attraction).

So we are explicitly violating point 2
(while we explicitly keep 1 and implicitly keep 3).

If the surface of the star deviates significantly from free-fall, then a large stress-energy builds up, which can force it further away from free-fall --- either stopping or exponentially delaying the collapse.

Precisely predicting what happens in a specific collapse scenario relies on messy model-dependent physics...

Collapse scenario:

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Many people are now (for numerous independent reasons) arguing against the standard Carter--Penrose diagram for the formation and evaporation of a semi-classical BH....

Apart from the nut-jobs (which we shall quietly discount),
there are hints from string-inspired models,
from attempts at unitarity preservation (in our asymptotic region...),
from one-loop curved-space QFT,
from analogue spacetimes,
all hinting at a more subtle history for collapse and evolution...

Specific predictions are frustratingly model-dependent,
but there is some "wriggle room" for interesting new physics...

A black hole for (almost) all practical purposes?

But some deep issues of principle remain...