Living on the edge: Cosmology on the boundary of anti-de Sitter space

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June 2000

Phys. Lett. **B482**, 183-194 (2000)

Abstract:

"Brane cosmology" is all the rage in the string theory/ quantum field theory/ phenomenology communities; generating theory papers at the rate of about 300 per year.

What's going on?

Why is it interesting?

- Physics.

- Sociology.

What are the prospects for an empirical test?

Overview:

"Brane cosmologies" are based on the idea that our familiar (3+1)–dimensional universe is a "membrane" (D-brane) in a more fundamental (9+1)–dimensional or (10+1)–dimensional reality.

These models are based on cross-fertilization between some of the more bizarre and outre speculative models of 20'th century physics.

Specifically:

- Mutant Kaluza–Klein theories.
  (Exotic Kaluza-Klein theories.)
- String theory/ D-brane theory/ M-theory/...
  (pick your favorite "nom du jour".)
- Feeble forces.
  (The "fifth force".)

Kaluza–Klein: I

The Kaluza–Klein theories date back to the early days of GR (1920's).

They started out as an attempt to classically unify electromagnetism with gravity, by introducing a "fifth dimension".

In the original versions this extra space dimension was supposed to be curled up into a little circle.

Electric charge was supposed to momentum in the fifth direction, with quantization of electric charge guaranteed by periodic boundary conditions in the extra dimension.

Being rather generous with the experimental bounds:

$$R_{extra\ dimension} < \frac{\hbar\ c}{1\ \text{TeV}} \approx 10^{-18}\ \text{metres.}$$

Kaluza–Klein: II

During the 1970's Kaluza–Klein theories got more complex — more dimensions, more classical fields (gluons, electro-weak bosons).

Fermions were added, as were various additional symmetries.

Almost always people kept the extra dimensions "compact" and small.

The suspicion generally was

$R_{extra\ dimension}$ < Planck length $\approx 10^{-35}$ metres.

This was the era of extended supergravity theories and the like.

By and large, these theories were not demonstrably wrong; but they were not demonstrably right either.

String theory: I

In 1984 Kaluza–Klein theories got a new lease on life with the advent of the anomaly-free string theories.

String theory had been languishing in the wilderness since the early 1970's, when it was first conceived as a model for the strong nuclear interactions, but quickly shown to have several undesirable features. (Mark 1 string theory.)

In the late 1970's an early 1980's steps were taken to recycle string theory as a candidate model for a theory of quantum gravity.

The discovery in 1984 that the resulting theory was free of quantum field theory "anomalies" surprised everyone.

(The point being that once you are sure that certain classical symmetries are *not* broken by quantum effects it is much easier to prove renormalizability or even finiteness.)

String theory: II

For various technical reasons string theory is most easily set up in $(9+1)$ dimensions, though $(10+1)$ dimensions is increasingly popular.

(There are also arcane ways of setting up string theory in $(3+1)$ dimensions, but let's not open that particular can of worms right now.)

If you are going to work in $(9+1)$ dimensions you had better quickly find a way of getting rid of the six extra embarrassing dimensions — Kaluza–Klein theories were drafted to do the job.

The resulting "Mark 2 string theory" is not demonstrably wrong; and is a reasonable candidate for a quantum theory of gravity; but is not demonstrably right either.

Brane theory: I

In the mid 1990's string theorists began to realize that string theory was not just a theory of strings but that it also contained a large number of membrane-like solitonic degrees of freedom that would be excited in generic collision processes.

Thus was born p-brane theory:

- -1 brane: instanton.
- 0 brane: particle.
- 1 brane: string.
- 2 brane: membrane.
- p-brane: p-dimensional object sweeping out a (p+1)-dimensional world volume.

Brane theory: II

In standard Kaluza-Klein theory the particles are free to move in the extra dimensions, which is why you have to keep them small.

In Kaluza–Klein theories based on brane theory, (Mark 3 string theory) string theory effects trap the matter on (near) the brane, and you can let the extra dimensions get large without mucking up particle physics.

Consider a 3-brane/D-brane (Dirichlet-brane); this means that the open strings in the theory are constrained to end on the D-brane.

If a string has energy E and string tension α, then the maximum distance it can penetrate into the extra dimensions is

$$L = \frac{E}{2\alpha}.$$

Brane theory: III

So the new model of empirical reality is this: We are living on a 3-brane in higher-dimensional spacetime.

All normal "particles" (electrons, quarks, photons) are associated with open strings whose end-points lie on the 3-brane, and are thus trapped on the 3-brane by stringy effects.

On the other hand, "gravitons" are associated with closed strings with no endpoints — they can in principle move off the 3-brane; and so you have a hybrid theory where (low-energy) particle physics is automatically (3+1) dimensional but gravity probes the higher-dimensional structure of "reality".

"For the weak shall overcome the strong..."

Feeble forces: I

Feeble forces (the "fifth force") are the theoretical framework used to describe hypothetical deviations from the inverse square-law of gravity.

Observationally, the inverse square law works well on solar system scales, and on stellar cluster scales.

Something goes wrong at galactic scales, but this is typically attributed to dark matter (not to a breakdown of the inverse square law).

The inverse square law also works well at laboratory scales (about 1 metre; Cavendish experiments).

Feeble forces: II

Surprise 1: Until about 1993 $G_{\text{laboratory}}$ as measured by Cavendish experiments could have differed from $G_{\text{solar system}}$ by up to 30 percent; present data constrain them to be equal to within one percent or so.

Surprise 2: Below about 5 cm, there's almost no constraints on the inverse square law.

This is one of the experimental windows that the brane theorists hope to exploit; the other is cosmology...

Brane model building: I

There are three main branches of brane-based model building:

- Antoniadis, Arkani-Hamed, Dimopoulos, Dvali (1998; 500 citations):
  In this scenario (AADD) the extra dimensions are large (millimetre) but still "compact" in the mathematical sense.

- Randall–Sundrum (1999; 300 citations):
  In (one version of) this scenario (RS1) the extra dimensions are large (millimetre) but still "compact" in the mathematical sense. We live on one 3-brane, and there is an additional anti-brane which carries "mirror matter".

Brane model building: II

- Randall–Sundrum (1999; 300 citations): In (one version of) this scenario (RS2) the extra dimensions are infinite and "non compact" in the mathematical sense. There is only one 3-brane and we are on it.

For technical reasons these models are all (4+1)-dimensional.

They start out in (10+1) dimensions, but six dimensions are eaten up in the usual Kaluza–Klein fashion with compact dimensions; it is only the last step in going from (4+1) to (3+1) dimensions that uses these "exotic techniques".

There are now also a number of papers attempting to have many infinite dimensions, not just one...

Brane model building: III

All three of these models predict deviations from the inverse square law at short distances, typically at centimetre scales.

The experimentalists are very happy to finally have some sort of prediction from string theory... (very happy!)

The other place where these brane models might lead to experimental/ observational testing/ verification are in cosmology...

Brane cosmology: I

I want to talk about a particularly simple and compelling implementation of brane cosmology.

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To keep life as simple as possible, I will have only one 3-brane, and only one bulk (4+1)-dimensional region.

The 3-brane will be the boundary of the (4+1)-dimensional region, so that we are quite literally "living on the edge" of spacetime.

Take the bulk geometry to be

$$\mathrm{d}s_{4+1}^2 = -F(r)\,\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{F(r)} + r^2\,\mathrm{d}\Omega_3^2.$$

$$\mathrm{d}\Omega_3^2 \equiv \mathrm{d}\chi^2 + \sin^2\chi\,(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2).$$

Brane cosmology: II

Now truncate this cosmology at finite "radius" $r = a(\tau)$.

The surface of this truncated geometry is automatically a (3+1)-dimensional $k = +1$ FLRW geometry with induced metric

$$\mathrm{d}s_{3+1}^2 = -\left[F(a(t)) - \frac{1}{F(a(t))}\left(\frac{\mathrm{d}a}{\mathrm{d}t}\right)^2\right]\mathrm{d}t^2 + a(t)^2\,\mathrm{d}\Omega_3^2.$$

Go to a new coordinate τ measuring proper time along the edge, then

$$\mathrm{d}s_{3+1}^2 = -\mathrm{d}\tau^2 + a(\tau)^2\,\mathrm{d}\Omega_3^2.$$

Brane cosmology: II

Now apply the (4+1)-dimensional Einstein equations. To do this you need a (4+1)-dimensional version of the "thin shell" formalism, modified for this one-sided brane. A brief agony of formal GR yields

$$8\pi\, G_{4+1}\, \rho_{3+1} = 3\, \frac{\sqrt{F(a)+\dot{a}^2}}{a}.$$

Rearranging

$$\left(\frac{\dot{a}}{a}\right)^2 = -\frac{F(a)}{a^2} + \left(\frac{8\pi\, G_{4+1}\, \rho_{3+1}}{3}\right)^2.$$

In contrast the *standard* Friedmann equation for a $k = +1$ closed FLRW universe is

$$\left(\frac{\dot{a}}{a}\right)^2 = -\frac{1}{a^2} + \frac{\Lambda}{3} + \frac{8\pi\, G_{3+1}\, \rho_{3+1}}{3}.$$

How do we get these equations to (more or less) agree?

Brane cosmology: III

Split the (3+1)-dimensional energy into a constant ρ_0 determined by the brane tension, plus "ordinary" matter ρ, with $\rho \ll \rho_0$ to suppress the quadratic term in comparison to the linear. Then

$$G_{3+1} = G_{4+1} \left(\frac{16\pi \, G_{4+1} \, \rho_0}{3} \right);$$

that is

$$G_{4+1} = \sqrt{\frac{3 \, G_{3+1}}{16\pi \, \rho_0}}.$$

Therefore

$$\left(\frac{\dot{a}}{a}\right)^2 = -\frac{F(a)}{a^2} + \left(\frac{8\pi \, G_{3+1}}{3}\right) \left[\frac{1}{2}\rho_0 + \rho + \frac{1}{2}\frac{\rho^2}{\rho_0}\right].$$

Observational cosmology forces us to take ρ_0 large [electro-weak scale or higher to avoid major problems with nucleosynthesis], and then forces us to deduce the presence of an almost perfectly countervailing cosmological constant in the bulk.

Brane cosmology: IV

For the (4+1)-dimensional Reissner–Nordstrom–de Sitter geometry

$$F(r) = 1 - \frac{2M_{4+1}}{r^2} + \frac{Q_{4+1}^2}{r^4} - \frac{\Lambda_{4+1}\, r^2}{6}.$$

M_{4+1} is a (4+1)-dimensional "mass" parameter, corresponding to the mass of the central object in (4+1)-space — it does not have a ready (3+1)-dimensional interpretation and is best carried along as an extra free parameter that from the 4-dimensional point of view can be adjusted to taste.

Q_{4+1} corresponds to an "electric charge" in the (4+1)-dimensional sense.

Our universe, the boundary D-brane, must then be viewed as carrying an equal but opposite charge to allow field lines to terminate.

Brane cosmology: V

¿From the (3+1)-dimensional view Q_{4+1} may be taken to be a second free parameter.

The (4+1)-dimensional cosmological constant combines with the term coming from the D-brane tension to give an effective (3+1) dimensional cosmological constant

$$\Lambda = \frac{\Lambda_{4+1}}{2} + 4\pi G_{3+1}\,\rho_0.$$

The generalized Friedmann equation is now

$$\left(\frac{\dot a}{a}\right)^2 = -\frac{1}{a^2} + \frac{2M_{4+1}}{a^4} - \frac{Q_{4+1}^2}{a^6} + \frac{\Lambda}{3} + \left(\frac{8\pi\, G_{3+1}}{3}\right)\left[\rho + \frac{1}{2}\frac{\rho^2}{\rho_0}\right].$$

By tuning these parameters appropriately one can recover standard cosmology to arbitrary accuracy.

Brane cosmology: VI

Rewrite the generalized Friedmann equation as

$$\begin{aligned}\left(\frac{\dot{a}}{a}\right)^2 &= -\frac{1}{a^2} + \frac{\Lambda}{3} + \left(\frac{8\pi\, G_{3+1}}{3}\right) \rho \\ &\quad + \frac{2M_{4+1}}{a^4} - \frac{Q_{4+1}^2}{a^6} + \left(\frac{8\pi\, G_{3+1}}{6}\right) \left[\frac{\rho^2}{\rho_0}\right].\end{aligned}$$

M_{4+1} can be used to mimic an arbitrary quantity of what would usually be called "radiation" (relativistic fluid, $\rho = 3p$).

Q_{4+1} mimics "stiff" matter ($\rho = p$), though with an overall minus sign.

An observational astrophysicist or cosmologist could now simply forget about the underlying string theory and D-brane physics, take this expression as the D-brane inspired generalization of the Friedmann equations, and treat M_{4+1}, Q_{4+1}, Λ, and ρ_0 as parameters to be observationally determined.

Prospects:

Of course we actually want to do more than just reproduce standard cosmology. We can already see several interesting possibilities:

- Look for short-distance deviations from the inverse square law.
- Use the extra free parameters M_4, Q_4, ρ_0, for better observational cosmological fits.
- Re-do the analysis for $k = 0$ and $k = -1$; (relatively easy but not as "nice").
- Re-do the analysis for more complicated (4+1)-dimensional geometries; (downright ugly and increasingly baroque).

Conclusions:

The model cosmology I've described in this talk is the best compromise I have come up with between something complicated enough to be interesting, and yet close enough to standard cosmology to not be completely unconstrained by current observational data.

If you actually want to confront string theory with observation or experiment, this sort of cosmology is the best hope string theory has.

Remember:

$$\begin{aligned}\left(\frac{\dot{a}}{a}\right)^2 &= -\frac{k}{a^2} + \frac{\Lambda}{3} + \left(\frac{8\pi\, G_{3+1}}{3}\right)\rho \\ &\quad + \frac{2M_{4+1}}{a^4} - \frac{Q_{4+1}^2}{a^6} + \left(\frac{8\pi\, G_{3+1}}{6}\right)\left[\frac{\rho^2}{\rho_0}\right].\end{aligned}$$

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